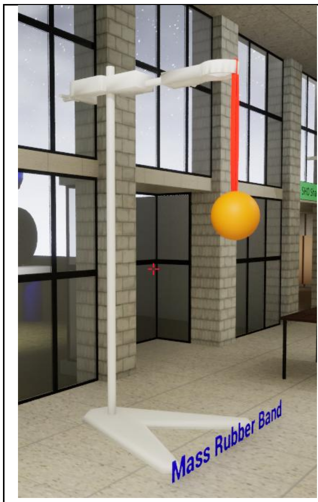
Chapter E

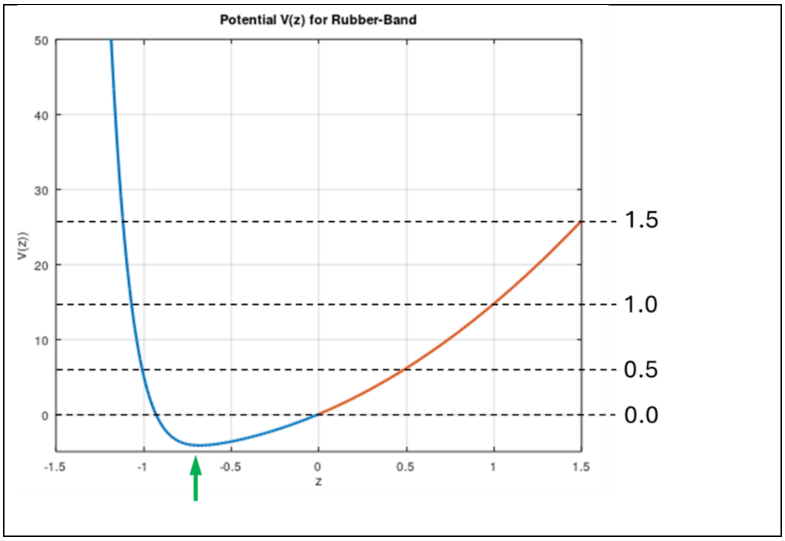
Mass on a Rubber Band

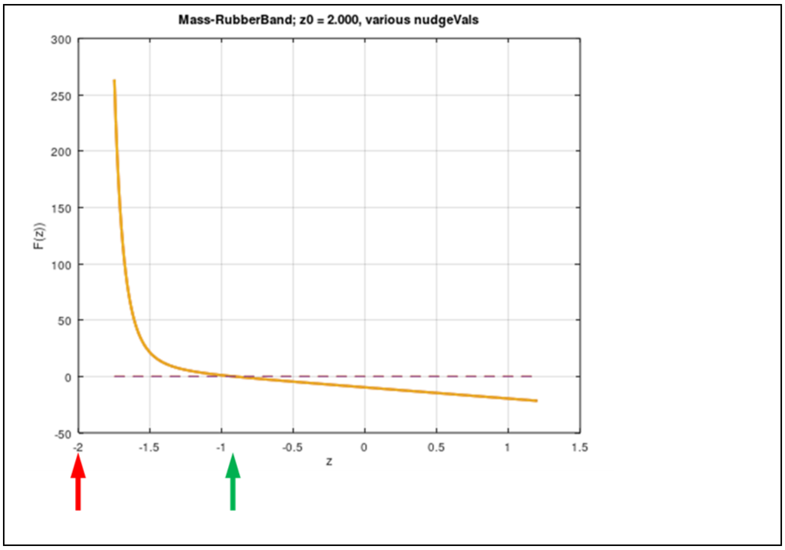
E.1 Summary of the Physics



Input from Teachers here

Perhaps include these plots.





E.2 PhysLab Experiments

ROADMAP

Overview of this chapter.

HOME EXPERIMENT

E1 Experience a Rubber Band

Find a rubber band and perform a ‘force-extension’ test. Slowly pull it, increasing the extension, and think about the force you feel. There will be a certain extension where the force really zonks up, rapidly increasing. You have just experienced the non-linear force when a rubber band is stretched a lot

DEMONSTRATION

E2 Observation of the Oscillator and its Time-Trace

We wish to see how the oscillator behaves, both by direct observation and also through inspection of its time-traces. The physics is clearly very different from the physics of harmonic oscillators, so we expect some pleasant surprises.

Here’s the steps you will need to do for this demonstration.

a) Select the apparatus and press **F** to load experiment **DEMO1**.
b) Press **F1** to start the experiment then **T** to apply an initial displacement.
c) Observe the motion of the mass and look for features which suggest it is not SHM.
d) Look at the time-traces, x-t at the top and v-t at the bottom. Can you see that these are not harmonic? How?
e) Look in detail at the v-t trace, especially the *rate of change of v* (which is acceleration). Can you identify times within each cycle where the average value is really different? Can you suggest what this means? Hint: If the accelerations are different then so are …
f) Press **Ctrl-Z** to display the phase plane. Look for a region where the velocity (vertical axis) is changing quickly. What is the value of z in this region? Explain what is happening here.
g) Press **F1** to pause then **P** to bring up the parameter menu and set **damping** to 0.5.
h) Press **F2** to reset then **T** then **F1** and observe the phase plane trajectory as the motion shows.
i) Convince yourself that for small amplitudes, the motion is almost harmonic. Explain how you convinced yourself.
j) Press **X** to disengage the apparatus.

DEMONSTRATION

E3 Plotting Oscillations and Interpreting Plots

Here you will collect and log data either for Octave or else Excel if you do not have Octave installed. Octave will automatically produce all the plots for you.

a) Select the apparatus, press **F** and load the experiment **DEMO1**.
b) Press **Z** to start recording then immediately press **T** then **F1**.
c) After 5 secs or so, press **Z** again to stop recording, then press **V** to write the data to file.
d) If you have Octave installed, press **O** to generate and show the plot files. If not, read below.
e) Press **X** to disengage with the apparatus.
f) If you are using Excel, we must create the plots manually, open the file **Mass_RubberBandX0.csv** (in the folder **ExcelFiles**).
g) Create a joint plot of displacement and velocity vs. time, and a second phase plane plot of velocity vs. displacement.
h) First look at the time-traces, convince yourself they do not show harmonic motion. What features of these plots support this?
i) What is the value of z near the 'sharp' troughs of the *z-t* curve? Where is the mass located at these places? Why is the curve 'sharp' at these places.
j) Look at the *v-t* curve and identify values of *z* where the rate of change of *v* (i.e. acceleration) is largest? What is the mass doing here? Explain what causes the acceleration to be large at these places.
k) Now look at the phase-plane plot. Can you find a segment which is almost an arc of a circle (suggesting harmonic motion)? Where is the mass at this place.
l) Where is the phase-plot trajectory definitely not circular (suggesting non-harmonic motion). What is the mass doing here?
m) You will notice a horizontal line on the phase-plot. The left end of this is the initial position of the mass. Using this point as reference, does the mass rise higher above

this, or lower behind this? Can you explain why there is this difference?

EXPERIMENT

E4 Small-Amplitude Oscillations

We know that the rubber band has a non-linear force-extension curve which leads to non-harmonic oscillations. However, theory predicts that if the oscillation amplitude is small, then the oscillations are *almost harmonic*, i.e. the time-traces will look *nearly* sinusoidal. That's the object of this experiment.

Data collection and Interpretation. Follow these steps to successfully collect some data.

a) Press **F** and select the experiment **Expt1**. This will displace the mass from equilibrium by 0.01m, a small amount.
b) Press **T** then **F1** and observe the time traces. Do these look approximately harmonic?
c) Press **Ctrl-Z** to bring up the phase plane. Does this correspond more-or-less to harmonic motion? Why is the closed curve located where it is?
d) Let's reset to capture some data. Press **F2** to reset, then
e) Press **T** then **F1** then **Z** to start data logging.
f) After around 5 secs, press **Z** to stop logging and **V** to write the log to disk.
g) If you have Octave installed press **O** to display the plots, else open the file **Mass_RubberBandX0.csv** in Excel and make separate plots of *z*-*t*, *v*-*t*, and *v*-*z*.
h) Explain how all three plots provide good evidence that small amplitude oscillations are approximately harmonic.

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INVESTIGATION

E5 Variation of Period with Amplitude

Since the rubber band is non-linear then we expect that the oscillation period will depend on its amplitude. Here you will investigate this. The term 'amplitude' is perhaps not the best, since the time-traces are asymmetrical. So we shall use the initial displacement from the equilibrium position as our independent variable.

(a) Investigation Planning. You need to choose a range of initial displacements. The independent variable is **nudgeHeight** which is the displacement from equilibrium (-0.684 m). Use the potential graph for guidance. Look at the range of initial values used there, that will be a good start. The starting value provided is -0.35. Before collecting data seriously, you could *play with the apparatus*; try out some values and see what happens.

(b) Data collection. You will use the **nudgeHeight** in the parameter menu to set the initial conditions.

a) Press **F** and load the experiment **Invest1**.
b) Press **F1** then **T**, observe the oscillations and note down the values of **nudgeHeight** and **period** from the HUD.
c) Press **P** to bring up the parameter menu, then reduce the value of nudgeHeight to your chosen value, I chose -0.30.
d) Press **F2** to reset to this value, then **F1** and **T**. Observe and note down the values of **nudgeHeight** and **period** from the HUD.
e) Repeat c) and d) for your chosen values.
f) Press **X** to disengage the apparatus.

(c) Data plotting and analysis

If you have Octave installed, open up the Octave script **Plotit_X.m** (in the **OctaveFiles** folder) and add the results for initial displacement and period. Else open up the Excel file **Plotit_X.xlsx**, add your data there, and then make a plot.

Inspect your plot and think about the following:

(a) Can you see an overall trend in your data?

(b) Is there any evidence of the period being roughly constant over any region of initial displacement?

(c) Could the plot be improved by more data collection? If so, then collect more data! Think carefully about which region of the plot you could best explore.

(d) Reflections and Formulation of conclusions.

You might like to think about the following:

(a) Can you suggest a reason why the period varies with amplitude as you have found?

E.3 Roundup

Here's the main ideas you should know:

(a) Rubber material has a non-linear force-extension graph: Under tension there is an extension above which the stiffness dramatically increases. Under compression there is no change in stiffness.

(b) The non-linearity can be explained by considering the rubber molecular structure.

(c) The nonlinearity results in nonharmonic oscillations, and the period of these is dependent on the amplitude.

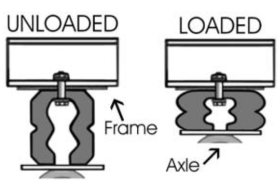
(d) For a non-linear system, small-amplitude oscillations are often approximately harmonic.

(e) Non-linear oscillations can be recognized by non-sinusoidal time traces, and a phase-plane curve which is not an ellipse or a circle.

E.4 Questions

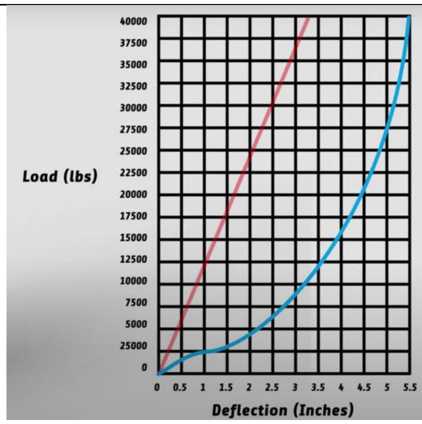
Input from Teachers here

1(S) Hollow rubber springs may be used in train and truck suspension, where the stiffness does increase with load. This is due to *changes in geometry* of the spring; at higher loads the hollow spring deforms, presenting a larger surface of rubber to hold the load. The diagram below illustrates this[1].



[1] Diagrams in this section are courtesy of Timbren.

The force-extension graph for such a spring is shown below (blue) compared with a coil spring (red). The data originates in the US, hence the 'Imperial' Units[2].

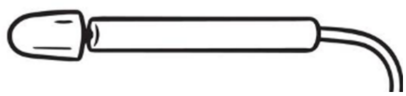


(a) Explain from the above graph how you can deduce that the stiffness of the hollow spring ultimately increases with deflection.

(b) For very large loads, which spring is stiffer, the coil or the hollow rubber? Explain.

(c) In an experiment with a truck fitted with hollow rubber springs if you measured the oscillations (deflection) following riding over a bump, what would the time-trace look like? How much non-linearity would you expect to see? Explain.

2(C) Think about a hand-held bike pump used as a spring



(a) Sketch the force-displacement curve, when you start with the plunger extracted and close off the pipe end, then slowly push the pump into the cylinder.

(b) Does this curve look familiar?

(c) Suggest how the shape of the curve results, drawing on appropriate physics.

[2] One inch is 2.54 cm, and one lb (pound) load is 4.45 Newtons.