Chapter D
Mass on a Spring

D.1 Summary of the Physics

Text based on the Instructors' Book stating major concepts and ideas without any proofs.

Input from Teachers here

Need to have the following theory.

$$z(t) = -\frac{mg}{k} + Ae^{-\beta t}\cos(\omega t), \qquad A = \frac{mg}{k}$$

$$\omega = \sqrt{(\omega_0^2 - \beta^2)}, \qquad \beta = \frac{b}{2m}$$

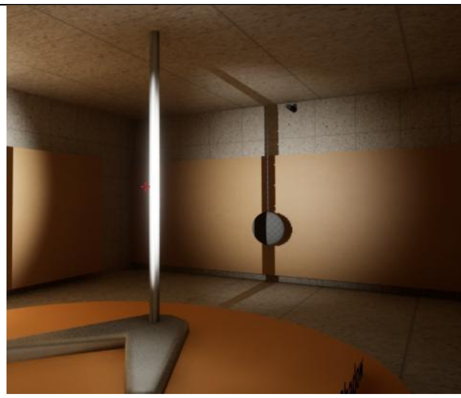
D.2 Roadmap

Here's an overview of the practical work in this chapter.

DEMONSTRATION

D1 Shadow Projection of Circular Motion

Here we discover a nice mathematical truth, but without details of the maths. A vertical pole rotates on a disk, and a light beam projects the shadow on a wall. This shadow appears to oscillate left and right. Also a pendulum is hung on the wall, the various coefficients have been set so the pendulum's motion is (more or less) the same as the shadow.



Here's how to start the demo

i. Find the apparatus **MAS22_SHO_Shadow** and press **F1** to start the engine.
ii. Observe the motion of the pendulum and shadow and convince yourself these are about the same.

Now, a little theory. When the pole rotates around the centre, the projection of the pole is proportional to the cosine or sine of the pole's angle. This is the shadow.

Use your observations to explain, if the angle of the pole at time *t* is$(2\pi/T_{pole})t$, where T_{pole} is the period of rotation of the pole in secs, then the location of the pendulum in the *x*-direction is given by $A\cos(2\pi t/T_{pole})$.

DEMONSTRATION

D2 Observation of Mass on a Spring with Gravity

A mass is attached to an unstretched spring. At first it will move down so the upward spring force balances the weight of the mass and stay there. Then you will apply a displacement. There is no damping.

(a) Find the apparatus **Mass_Spring** and select it (cross-hair, left-click).
(b) Press **F** and select Demo1. Hit Done.
(c) Press **F1** to start the experiment. The mass will move to its equilibrium position and stay there.
(d) Press **T** to apply an upward displacement of 0.5 m.
(e) Observe the oscillations, and the time-traces (displacement at the top, velocity at the bottom). Wait until the graphs run over two complete plots.
(f) Press **Ctrl-Z** to switch the graph to the phase plot. Wait until the plot is complete.
(g) Press **Ctrl-Z** to return to the time-traces. Wait a while.
(h) Press **X** to disengage the apparatus.

Here's some questions for you. You might like to re-run the experiment to help you find answers.

(a) When the mass has come to rest, what are the two static forces acting on it?

(b) Use your answer to (a) and a bit of the above theory to calculate the value of z when the mass is in equilibrium.

(c) Compare your answer to (b) with the equilibrium value of displacement from the HUD.

EXPERIMENT

D3 Changing Parameters

Here you will run a different experiment where the mass starts at its unstretched length and is not set to its equilibrium position.

(a) Select the experiment **Demo2** and press **F1** to run and observe the behaviour of the oscillator.
(b) Use **Ctrl-Z** to check out the time-traces and phase plot.
(c) Note down the **period** of oscillation from the HUD.
(d) Press **P** to bring up a list of parameters and change the value of **mass**. Observe how this affects the mass (there are two changes).
(e) Note down the mass and the period.
(f) Repeat (d) – (e) for several values of mass.
(g) Restore **mass** to 1.0 and now make changes to the spring stiffness. Remember to note down the stiffness and period.

Here's some questions for you:

(a) Make a couple of quick calculations to check your values of mass and period agree with the formula for period.

(b) Repeat for the spring stiffness.

(c) Increasing the mass changes the oscillation period but makes a second change. Explain this second change.

EXPERIMENT

D4 Effects of Damping

Here you will run the same experiment but now with damping
Here's the steps you need:

(a) Select **Demo1** and press **F1** then **T** to get the pendulum moving.
(b) Now press **P** to get the parameters and set **damping** to 0.25 and observe the oscillator and its time-traces.
(c) Reset the experiment to its new parameters and run again: press **F2** then **F1** then **T**. When running press **Ctrl-Z** to view damping in the phase plane.
(d) Press **P** and set **damping** to 0.5. Press **F2, F1, T** and observe the oscillator. Check out both time-traces and the phase plot.

(e) Repeat for a damping of 1.0, or anything else you fancy.
(f) Press **X** to disengage the apparatus.

Here's some questions for you:

(a) Look at the time traces of displacement and velocity. Describe how these differ from the time traces without damping.

(b) Explain what has caused these traces to be different.

(c) From these traces suggest when the mass comes to a complete rest.

(d) Look at the trajectory on the phase plane. How would you describe this curve?

(e) Explain why this curve has the shape it does have.

(f) Continue plotting this curve in your mind and work out the coordinates where the mass will be at rest.

EXPERIMENT

D5 How the spring constant affects the oscillation period

The period of oscillation is a very important property of an oscillator. Think of an oscillator used as a clock; its period should be and remain at exactly 1 second. The expression for the period of an undamped oscillator is $T = 2\pi\sqrt{m/k}$ so you see it depends on both the mass of the bob and the stiffness of the spring. Here we shall focus on the stiffness *k*. You will collect some data and then plot it using Octave or Excel.

(a) Data collection. Follow these steps to successfully collect some data.

(a) Select the apparatus and the experiment **Expt D5**
(b) Press **P** and set the **stiffness** to 50 N/m.
(c) Press **F1** and after some time note down the period and the amplitude from the HUD. (Make sure the graph is set to time-traces).
(d) Press **P** to set the **stiffness** to 40 N/m and after some time note down the period and amplitude from the HUD
(e) Repeat pressing **P** to reduce the stiffness until you have collected data for stiffness down to 5 N/m.

(f) Press **X** to leave the apparatus

(b) Data plotting and analysis

Theory shows that the period is related to the spring stiffness k by the expression

$$T = 2\pi\sqrt{\frac{m}{k}}$$

(a) Either

(i) If you have Octave installed, enter your data into the script **Plotit_1.m** (in the folder **UserOctave)** and run the script in Octave. Your data points will be plotted on a curve drawn from the above theory.

(ii) Else open up the file **Plotit_1.xls** Excel (in the folder **UserExcel**) and enter your data.

(b) Comment on the agreement between your results and theory.

(c) A straight line is always the best way to check theory vs. experiment. How would you do this?

INVESTIGATION

D6 Measuring the effects of Damping.

We know that damping reduces the amplitude of oscillation over time, and the expression for this is

$$z(t) = Ae^{-\beta t}, \qquad \beta = b/2m$$

This shows us that the amplitude (in theory) never reaches zero, the reduction in amplitude gets smaller with time. Note that this solution corresponds to setting $g = 0$ in our model.

We wish to measure the time needed for the amplitude to reduce to 10% of the starting amplitude. So from the above expression we have

$$0.1 = e^{-\beta t}, \quad i.e., \qquad \log 0.1 = -\beta t$$

So the time taken to reduce to 10% of the initial amplitude is just

$$t = -\frac{\log 0.1}{\beta}$$

In this investigation you will measure this time for various damping values and compare with the above theory. Remember that $\beta = b/2m$ and b is the value of the damping parameter.

(1) Investigation Planning. Here we shall take the approach of starting the mass with a displacement of 0.5m and noting the time, from the HUD, where the displacement drops just below 0.05m.

You will need to collect at least 5 measurements. The question is, what values of damping *b* should you use? The best way forwards is to *play with the apparatus* and try various values of *b* perhaps starting at 0.5 and increasing to 1.0 or larger. Then decide on a series of *b* values which you will use to collect data.

(2) Data collection. Follow these steps to successfully collect some data.

(a) Select the apparatus and press **F** and select experiment **Invest_D6.**
(b) Press **Alt-H** to highlight the HUD values.
(c) Press **P** and set **gravity** to zero and keep it there. Also set your first value of **damping**.
(d) Press **F2** to reset the oscillator to your parameters.
(e) Press **T** then **F1** to start the experiment. Heep an eye on the HUD **maxDispZ** and press **F1** when this reaches 0.05. Note down the **time** from the HUD and your value of damping.
(f) Press **P** to set your next value of **damping** then
(g) Repeat (e) – (f) for your set of damping values.
(h) Press **X** to leave the apparatus.

(3) Data plotting and analysis

You can enter your date into the file **Plotit_2.xls** or **Plotit_2.m** depending on whether or not you are using Octave. Here's some questions to answer looking at your plot.

i. Do your results broadly agree with the theory curve or are there errors?
ii. If there are errors, are these *systematic* (e.g., all your points follow the curve but are too high or too low)? or are they *random* (some above and some below the curve)?

iii. Having identified the type of errors (if there are any) could you attempt to explain why? You should think about possible limitations in the way you collected data.

iv. You may decide to run more experiments to fill-in any 'holes' in your plot.

(4) Reflections and Formulation of conclusions.

You might like to write a few words in simple English to explain what damping does, and what are the features of harmonic oscillations with damping.

INVESTIGATION

D7 Does Amplitude affect Period

The theory of harmonic oscillators tells us that the changing the oscillation amplitude never ever changes the oscillation period. Here you are invited to investigate whether or not this is true for the mass-spring oscillator.

(a) Investigation Planning. By now you will have enough experience with the apparatus to make a straightforward plan of changing the amplitude and measuring the period. Also you will be able to work the engine to collect results using the HUD.

You will need to decide if you want to plot your data, if so decide *how* to plot it. You might like to calculate percent errors between experiment and theory, here's how to do it

$$\%error = 100\frac{T_{Expt} - T_{Theory}}{T_{Theory}}$$

A good starting point is to calculate the period expected using the theory provided.

D.3 Roundup

Here's the main ideas you should know.

(a) Harmonic oscillators have period independent of amplitude

(b) The period of the mass-spring oscillator is related to the spring stiffness by the relation $T = 2\pi\sqrt{m/k}$

(c) Damping produces an exponential decay in amplitude of the form $Ae^{-\beta t}$ and the amplitude tends to zero as $t \to \infty$.

(d) The phase plane shows undamped oscillations as circles, and damped oscillations as inward spirals.

(e) Oscillation displacement is equivalent to a projection of circular motion.

D.4 Questions

These are labelled short (S), long (L) or conceptual (C) where you are not asked to perform calculations.

Input from Teachers here

1(C) A mass is hung vertically from a spring in a lab on Earth. The apparatus is taken into deep space where there is no gravity. Will it oscillate?

2(C) In your laboratory you have a collection of identical masses and identical springs. One oscillator is made from one mass suspended from one spring. You are asked to build a second oscillator with identical period with two masses. How would you do this?

3(C) You have a mass on a spring which oscillates. If the mass of the object is multiplied by four, how must you change the oscillation amplitude if the maximum speed of the mass is to be the same?

4(C) This question is about car suspension. To support a large load the spring stiffness k needs to be large. To have a comfortable ride the stiffness needs to be small to ensure a large period of oscillation. How might this conflict be resolved? [Hint, you might consider the unstretched length L_0 as a design parameter.

5(S) A mass of 0.001 kg is suspended from a spring and set in oscillatory motion. At $t = 0$, the displacement is found to be 0.5m and the acceleration -0.01 m/s/s. Find the stiffness of the spring.