Chapter X
Student Questions

X.1 A Brief Introduction

These questions are designed to bridge the gap between sixth form and 1st-year university physics courses. There are three sections. **Short Questions** have little or no guidance and are more suited to undergraduates. **Long Questions** present identical material but with intermediate stages to guide the student. These are perhaps more suited to students at school. While both these question types emphasize mathematics as the “language of physics”, the third section **Conceptual Questions** are more open-ended and focus on underlying physical concepts and perhaps experience, though mathematics is still important here. They are broad in scope and not limited to oscillations and waves.

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X.2 Shorts

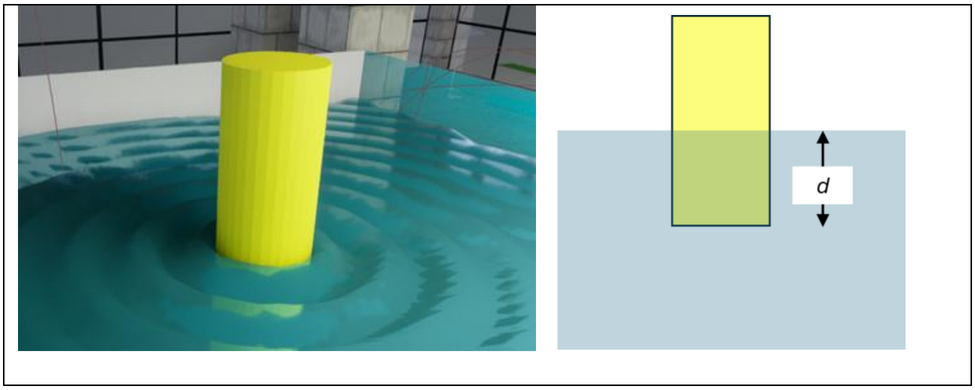
QS1 A floating cylinder

A vertical cylinder is floating at equilibrium in a water tank with draught d, the submerged length of the cylinder.

(a) Derive an expression for the frequency of oscillation of the cylinder.

(b) Experiments measure a frequency slightly smaller than predicted. Suggest what could cause this effect.

(c) When the cylinder oscillates it produces waves as shown in the figure. Suggest how these may affect the oscillations of the cylinder.

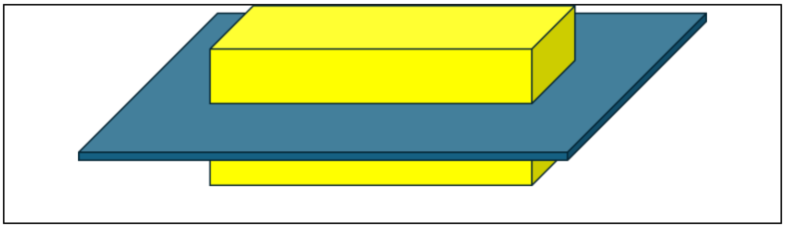


QS2 Design of a barge

A sea-going barge can be modelled as a rectangular hull with width and breadth. It is required to transport a given load (mass). As floating objects, they have a natural frequency of oscillation. To avoid large stresses on towing cables, this frequency should not coincide with the frequency of the waves they encounter (say 0.25 Hz), avoiding resonance.

(a) Derive an expression for the frequency of oscillation of a barge in terms of its principal parameters.

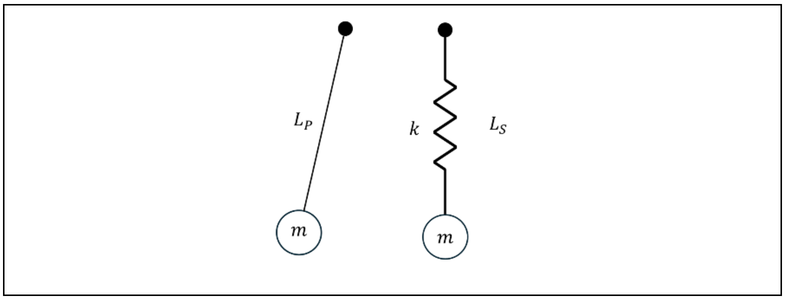
(b) Suggest the size of a barge if it is to avoid resonance.



QS3 Mass-spring and Pendulum

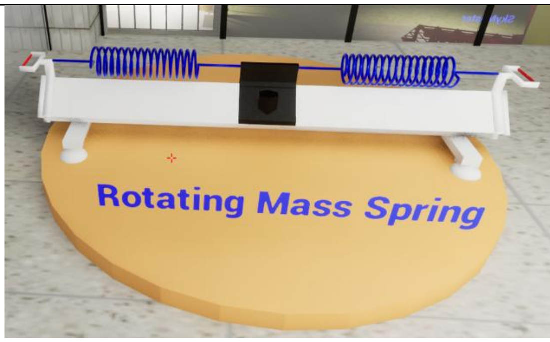
A pendulum with length L_P and bob of mass m is suspended next of an identical mass attached to a spring of stiffness k. The spring has length L_S when the mass is attached and has unstretched length $L_S^{(0)}$.

How would you choose the pendulum's length, so it has approximately the same oscillation period as the spring?



QS4 Rotating Air Track (1)

A glider moves along an air track constrained by two springs of equal stiffness and is free to oscillate along the track. The air track is itself is mounted on a platform which can rotate with an angular speed Ω rad/sec. Assume there is no friction.



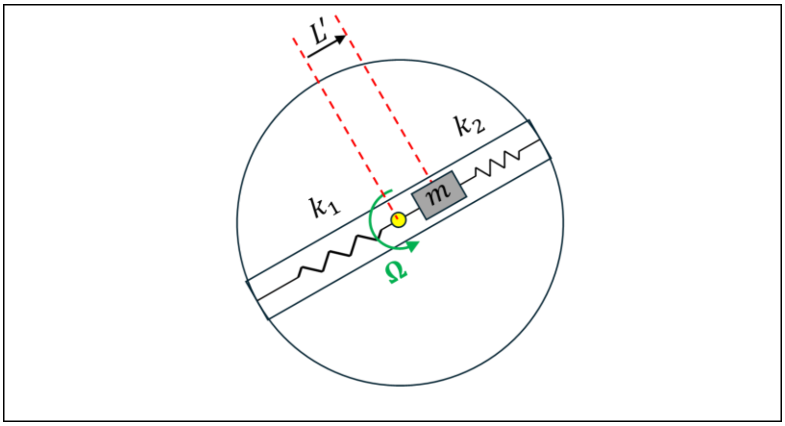
Explain how the frequency of the glider oscillations changes as Ω is slowly increased from zero and make a sketch of the relationship.

Show that there is a critical rotation speed Ω_c where the glider does not oscillate. What is its motion at this speed?

QS5 Rotating Air Track (2)

A glider mass m moves along an air track constrained by two springs of stiffness k_1 and k_1 and is free to oscillate along the track. The air track is itself is mounted on a platform which can rotate with an angular speed Ω rad/sec. The diagram below shows the glider at its equilibrium position (no oscillation, no rotation), it is located at distance L' from the centre of rotation.

(a) Write down the equation of motion for the glider about its equilibrium position (second order differential equation) assuming there is no friction.



(b) Obtain a solution assuming the glider starts with initial conditions $x(t = 0) = x(0)$ and $\dot{x}(t = 0) = 0$, and describe the main features of the glider behaviour.

(c) Show that there is a critical value of Ω where the glider will not oscillate. Describe the features of the glider behaviour as this value is approached.

QS6 Rotating Pendulum (1)

A pendulum length L is placed on a rotating platform with its support point along the axis of rotation of the platform. The pendulum is constrained to move in a vertical plane which rotates with the platform. The rotational angular speed of the platform is Ω.

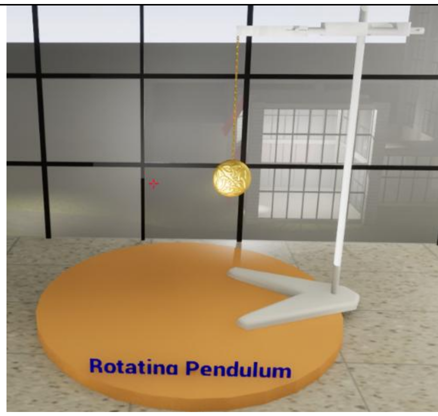
(a) Show that for *small angles of displacement*, the pendulum will oscillate and when rotating its angular frequency will be less than $\sqrt{g/L}$ and that for a certain critical rotation speed Ω_{crit} its frequency drops to zero. Make an annotated sketch how its frequency changes with Ω.

(b) Show that when $\Omega > \Omega_{crit}$ then the pendulum can have a stable non-vertical equilibrium and derive an expression for the angle of this.

(c) Show that the pendulum can oscillate around this new equilibrium and derive an expression for the frequency of oscillation.

QS7 Rotating Pendulum (2)

A pendulum length L is placed on a rotating platform with its support point offset from the axis of rotation of the platform. The pendulum is constrained to move in a vertical plane which rotates with the platform. The rotational angular speed of the platform is Ω.



(a) Write down the equation of motion of the pendulum (2nd-order differential equation) assuming there is no friction. And simplify it using the small-angle approximation $\sin\theta \approx \theta$ and $\cos\theta \approx 1$.

(b) Deduce how the angular frequency of pendulum oscillation ω depends on platform rotation speed Ω. Make an annotated sketch of this relationship labelling the important features.

(c) Assume a solution of the form $\theta(t) = A\cos(\omega t + \varphi)$ and initial conditions $\theta(t = 0) = \theta(0)$ and $\dot{\theta}(t = 0) = 0$, derive an expression for angle as a function of time.

(d) Using your expression in (c) explain how the offset influences the pendulum's oscillatory behaviour.

QS8 Car Suspension

A mass is attached to a spring pf uncompressed length L_0 and the spring shrinks to length L.

(a) Find an expression for the period of oscillation of the mass and show that it does not depend on the mass.

(b) Typical Land Rover springs have an uncompressed length of 54 cm and when installed have a compressed length of 34 cm. Estimate the period of oscillation of a Land Rover.

QS9 Cubic Oscillator

The equation of motion of a particular cubic oscillator is

$$m\ddot{x} = kx - x^3$$

Show that there exist small-amplitude oscillations around the stable equilibrium points with frequency

$$\omega = \sqrt{\frac{2k}{m}}$$

QS10 Double-well Oscillator

A system which supports oscillations is described by the following potential

$$V(x) = \frac{1}{4}(x^2 - k)^2$$

(a) Make a sketch of the potential well and annotate this with important features.

(b) Determine the frequency of oscillation of the system around the bottom of the wells.

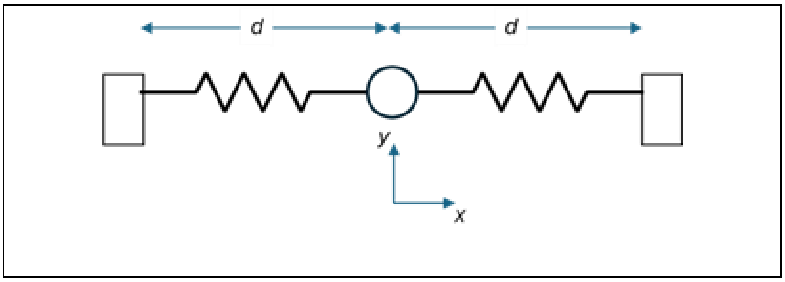
(c) If the potential were modified by addition of a term $+\delta x$ where $\delta < k$ describe the effect on the sketch of the wells and suggest how this might change the frequencies.

QS11 Springs and Stiffness

A mass m is suspended from a spring of stiffness k. The top of the spring is attached to a plastic ruler. When a force F is applied to the ruler, it deflects downwards by a distance αF. Find an expression for the frequency of oscillation of the mass. How can this be simplified if (i) the ruler is very stiff, (ii) the spring is very stiff?

QS12 Transverse Mass – Springs

A mass m is tethered by two springs each of stiffness k and unstretched length L_0 and is free to move on the horizontal plane.



The unstretched length of the springs are less than d so the springs are in tension.

(a) Derive an equation of motion for the mass in the y-direction and show that if $y \ll d$ then the mass can show harmonic motion.

(b) Derive an expression for the frequency of oscillation

QS13 Water wave Dispersion

The properties of travelling waves in water depend on the depth of the water h. Waves of angular frequency ω and wavelength λ obey the *dispersion relation*

$$\omega^2 = gk \tanh kh$$

where $k = 2\pi/\lambda$.

(a) Derive an expression for ω^2 for *deep* water waves, where $kh \gg 1$.

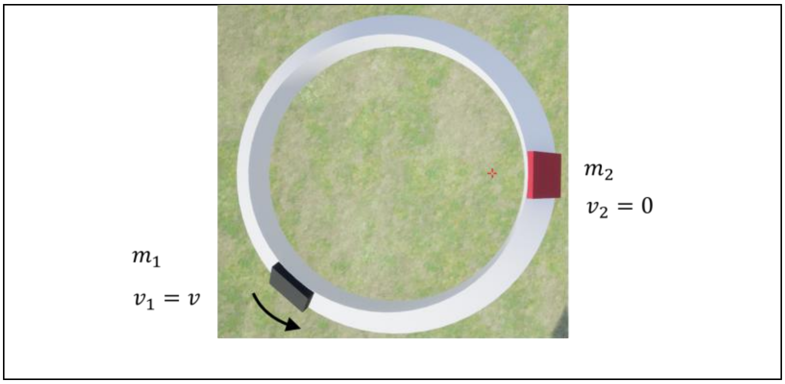
(b) Typical ocean waves in deep water have a period of around 4.0 secs. What wavelength would you expect to observe?

(c) If you had to solve the exact dispersion relation numerically, by writing a computer program, suggest a way of doing this.

$$\boxed{\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}}$$

QS14 Circular Air Track

Two gliders move on a frictionless circular (hypothetical) air track. Glider-2 starts at rest and glider-1 starts with velocity *v*. All collisions are elastic.



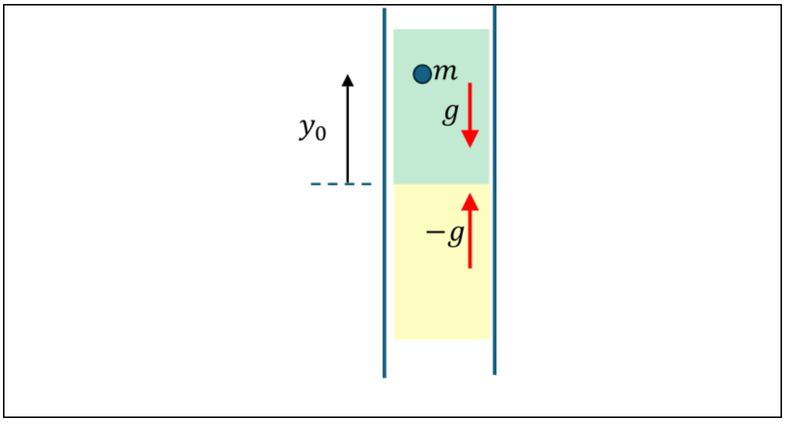
(a) Consider the case $m_2/m_1 = 3$. How do the velocities of the gliders change with time?

(b) Derive an expression for the velocity changes at the i'th collision for the general case $m_2/m_1 = \mu$.

(c) Find values for μ where there is a significant change in glider behaviour.

QS15 Gravity Tube

A vertical tube contains a top region where gravity g is pointing downwards and a lower region where the gravity is also g but pointing upwards. A small mass m is released at a height y_0.

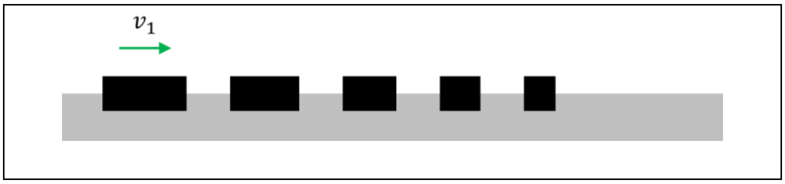


(a) Show that the mass executes harmonic motion, but that this is not simple harmonic motion.

(b) Derive an expression for the period of oscillation and determine which parameter(s) this depends on.

QS16 Collision Cannon

A series of gliders of progressively decreasing mass are placed on a frictionless air track. All gliders are at rest except the most massive which starts with velocity v_1.



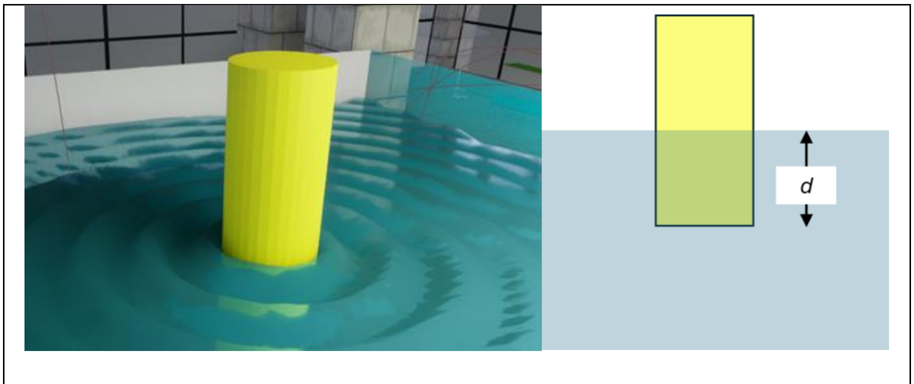
(a) If the mass ratio between successive gliders is μ, where $\mu < 1$, find an expression for the velocity of the right-most glider if there are N gliders in total.

(b) What is the maximum value of the total velocity magnification?

X.3 Longs

QL1 A floating cylinder

A vertical cylinder with mass m and cross-sectional area A is floating at equilibrium in a water tank with draught d, the submerged length of the cylinder. The density of water is ρ and the acceleration of gravity is g.



(a) Think about the forces on the cylinder when it is floating in equilibrium and write down an expression showing the balance of upward and downward forces on the cylinder

(b) Now the cylinder is pushed downwards a distance z. Write down an expression for the *restoring* force on the cylinder.

(c) [Look at the Background Info] Use your answer to (b) to complete this expression for the equation of motion of the cylinder, where you must fill in the [...]

$$ma_z = -[\ldots]z$$

(d) Now derive an expression for the angular frequency of oscillation.

(e) Look at the form of your expression, it should look familiar. Think about other oscillating systems you have encountered, a mass on a spring or a pendulum, or something else. See any similarity?

Background Info. In a situation where the restoring force on a mass is proportional to displacement z, then the equation of motion of the mass is, where a_z is the acceleration of the mass in the z-direction,

$$ma_z = -[\ldots]z$$

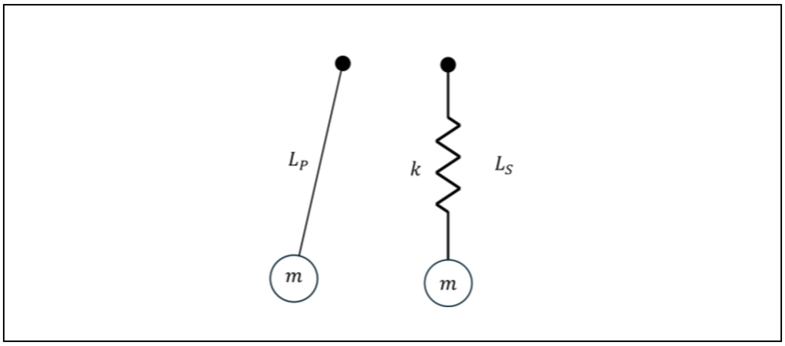
Here [...] is a load of parameters for the specific problem. The solution of this is harmonic motion with angular frequency[1] ω

$$\omega = \sqrt{[\ldots]/m}$$

[1] This is related to the frequency f in Hertz by $\omega = 2\pi f$.

QL3 Mass-spring and Pendulum

A pendulum with length L_P and bob of mass m is suspended next of an identical mass attached to a spring of stiffness k. The spring has length L_S when the mass is attached and has unstretched length $L_S^{(0)}$.



(a) Write down an expression for the angular frequency of oscillation of the pendulum.

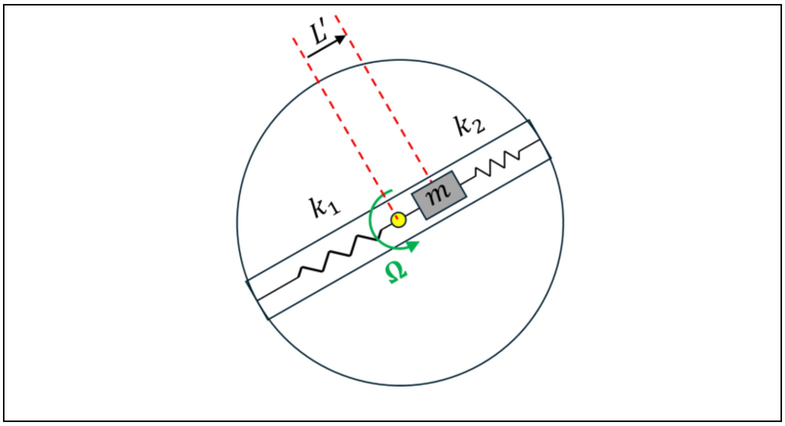
(b) Repeat for the mass-spring.

(c) Starting with an unloaded spring, the mass is added and the system is at equilibrium. Use this equilibrium to derive an expression for k/m including unstretched and stretches spring lengths.

QL5 Rotating Air Track (2)

A glider mass m moves along an air track constrained by two springs of stiffness k_1 and k_1 and is free to oscillate along the track. The air track is itself is mounted on a platform which can rotate with an angular speed Ω rad/sec. The diagram below shows the glider at its equilibrium position (no oscillation, no rotation), it is located at distance L' from the centre of rotation.

(a) Write down the equation of motion for the glider about its equilibrium position (second order differential equation) assuming there is no friction.



(b) Assume a solution of the form

$$x(t) = A\cos(\omega t + \varphi) + C$$

And substitute into the equation of motion.

(c) Assuming the glider starts with initial conditions $x(t=0) = x(0)$ and $\dot{x}(t=0) = 0$, show that $\varphi = 0$ and derive expressions for ω, A, C.

(d) Now write down the solution $x(t)$.

(e) From your results to (c) show that there is a critical value of Ω where the glider will not oscillate.

(f) Explain how the amplitude and location of the glider change as the rotation speed approaches the critical value.

QL6 Rotating Pendulum

A pendulum length L is placed on a rotating platform with its support point along the axis of rotation of the platform. The pendulum is constrained to move in a vertical plane which rotates with the platform. The rotational angular speed of the platform is Ω.

(a) Derive the equation of motion for the rotating pendulum (2^{nd}-order differential equation).

Now let's consider *small angles of displacement* of the pendulum.

(b) Using the simplifications $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ for small angles derive a simplified equation of motion.

(c) Use this to write down an expression for ω the angular frequency of oscillation.

(d) From your result in (c) show that there is a critical value of Ω where the oscillation frequency drops to zero. Sketch how ω depends on Ω.

(d) Assuming a solution of the form $\theta(t) = A\cos(\omega t + \varphi)$ and initial conditions $\theta(t = 0) = \theta(0)$ and $\dot{\theta}(t = 0) = 0$, derive an expression for angle as a function of time.

Now let's consider *large angles of displacement* of the pendulum.

(e) Return to your equation of motion in (a) and solve this for non-zero equilibrium angles θ_{equ} as a function of Ω. Sketch this solution.

(f) From (e) deduce a condition on Ω to allow the existence of these solutions.

(g) Derive an expression for the angular frequency of oscillation ω around the non-zero equilibrium angles.

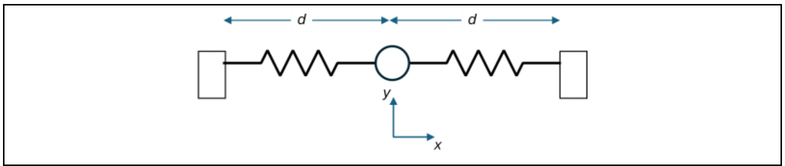
Background Info. You can derive ω from

$$\omega = \sqrt{-\frac{d\ddot{\theta}}{d\theta}}$$

where the derivative is evaluated at θ_{equ} .

QL12 Mass and Springs (1)

A mass m is tethered by two springs each of stiffness k and it is free to move in the horizontal plane as shown below.



The initial length of each spring is L_0 which is less than the spacing d, so the springs, as shown above, are in tension.

(a) Show that the equation of motion of the spring is

$$m\frac{d^2y}{dt^2} = -2k\left[1 - \frac{L_0}{\sqrt{d^2 + y^2}}\right]y$$

and explain how this shows the motion is not simple harmonic.

(b) Now take the case where the initial displacement y is small, i.e. $y \ll d$. Insert this condition into the above equation and show the resulting equation demonstrates simple harmonic motion.

(c) Find an expression for the period of this motion.

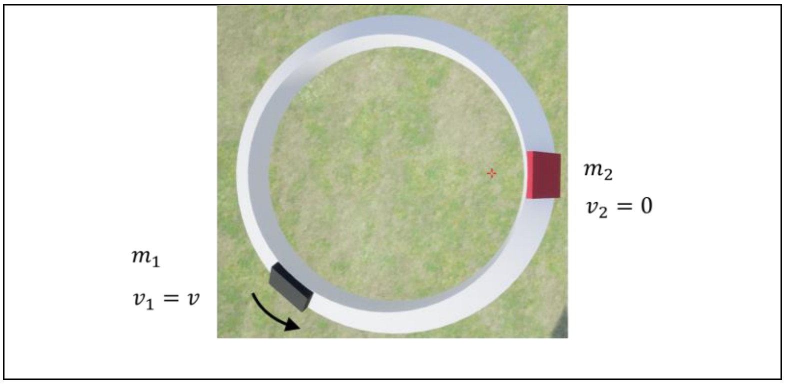
(d) Now assume that $L_0 \ll d$. What does this mean physically? Think about how tautly the spring is stretched. Derive an expression for the period under these new conditions.

(e) Consider longitudinal oscillations in the horizontal x-direction. Following a similar approach as above, derive an expression for the period of oscillation.

(f) Derive an expression for the ratio of periods along the x and y directions. Do not assume $L_0 \ll d$. How must the system be configured to make these two period almost the same?

QL14 Circular Air Track

Two gliders move on a frictionless circular (hypothetical) air track. Glider-2 starts at rest and glider-1 starts with velocity v. All collisions are elastic.



(a) With the following ratio $\mu = m_2/m_1$ derive an expression for the velocities after a single collision

(b) Show that the velocities before and after the i'th collision can be expressed as

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}^{(i+1)} = \underline{A} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}^{(i)}$$

where the matrix is

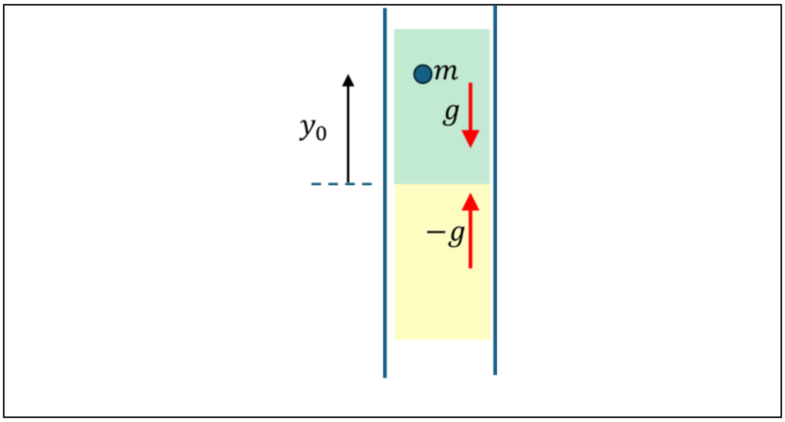
$$\underline{A} = \frac{1}{1+\mu} \begin{bmatrix} (1-\mu) & 2\mu \\ 2 & -(1-\mu) \end{bmatrix}$$

(c) Calculate the velocity transformation for $\mu = 3$. What does this tell you about the motion of the gliders?

(d) Show that the square of matrix $\underline{A}$ is the identity matrix. What is the physical significance of this?

QL15 Gravity Tube

A vertical tube contains a top region where gravity g is pointing downwards and a lower region where the gravity is also g but pointing upwards. A small mass m is released at a height y_0.



(a) Use the appropriate equation of motion with constant acceleration to find an expression for t, the time it takes for the mass to reach $y = 0$ (dashed line)

(b) Find an expression for the time it takes for the mass to come to rest in the lower region of gravity.

(c) Deduce an expression for the period T of the oscillations.

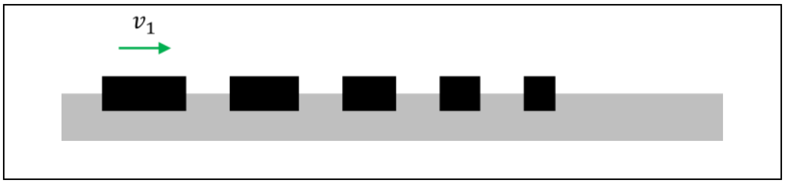
(d) Sketch a graph of the position $y(t)$ of the mass as a function of time and add labels to this graph.

(e) Repeat (d) for the velocity of the mass.

(f) Explain in simple and succinct English why the oscillations are not simple harmonic.

QL16 Collision Cannon

A series of gliders of progressively decreasing mass are placed on a frictionless air track. All gliders are at rest except the most massive which starts with velocity v_1.



If the mass ratio between successive gliders is μ, where $\mu < 1$, find an expression for the velocity of the right-most glider if there are *N* gliders in total.

(a) Consider two gliders, m_A moving with velocity *v* and m_A at rest. If the velocities after a collision are v_A and v_B then show that conservation of momentum and energy lead to the following expressions,

$$u = v_A + \mu v_B$$

$$u^2 = v_A^2 + \mu v_B^2$$

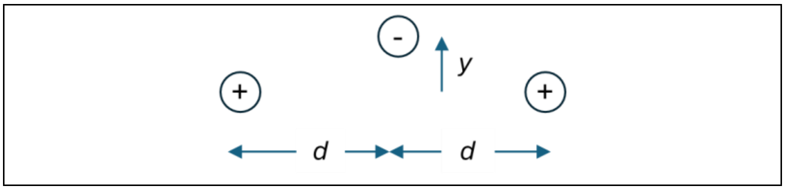
(b) Solve these for the ratio v_B/v_A.

(c) Now apply your result to the case of *N* collisions.

(d) What is the limit of the total velocity magnification as $\mu \to 0$?

QL17 Three Interacting Particles

Two positive particles and a negative particle are arranged as shown below with a total separation *2d*. The negative particle is constrained to move in the y-direction.



Assume that electrostatic forces follow the following law where r is the particle separation.

$$F(r) = \pm \frac{K}{r^2}$$

(a) Derive an expression for the force on the negative particle as a function of y.

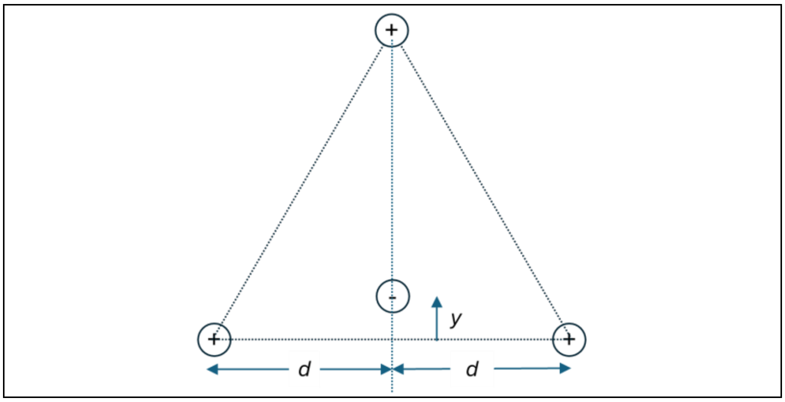
(b) Derive the differential equation of motion of the particle in the y-direction. Assume there is no damping.

(c) What particle behaviour does this equation suggest.

(d) For small particle displacements, $y \ll d$ derive an expression for the period of oscillation of the particle.

QL18 Four Interacting Particles

Three positive charges are located at the vertices of an equilateral triangle of side *2d* as shown below. A third negative particle, constrained to move in the *y*-direction is located as shown below



(a) Derive an expression for the force on the negative particle when it is displaced a distance *y* as shown.

(b) Check your expression predicts the correct force when the negative particle is located equidistant from the positive particles.

(c) Complete the Octave script **FourParticles.m** to plot out the force as a function if displacement *y* (for give values of *K* and *d*).

(d) Look closely at the plot, you should see two displacements where the force is zero. One of these is expected (why?) the other perhaps not.

(e) Look at the equilibrium points found in (d). By considering a small shift in *y* about each point (and looking at the resulting force change), explain why one of these equilibrium points is stable and the other is unstable.

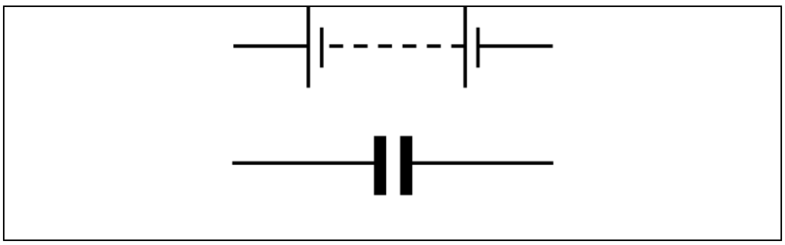
(f) Plan and conduct an investigation of oscillations around the stable equilibrium as a function of initial particle displacement.

X.4 Conceptual Questions

QC1 Beer glass Bubbles. In a glass of beer bubbles can be seen to accelerate upwards and reach a terminal velocity moving upwards. Therefore bubbles have negative mass. Discuss.

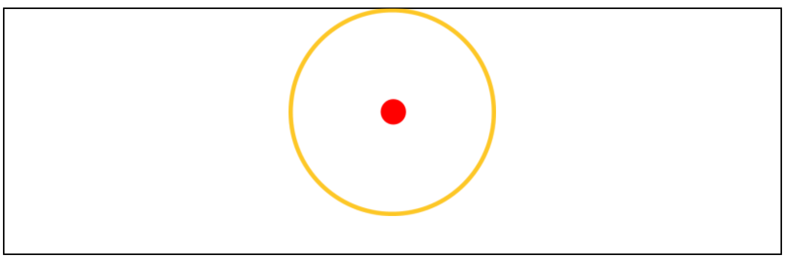
QC2 Car Suspension Design. Coil springs are often used in car suspension. To support the car's weight the stiffness of the springs needs to be large. To provide a smooth ride (with large oscillation period) the stiffness needs to be small. Discuss this apparent contradiction.

QC3 Electric Field. A capacitor is placed close to a battery as shown. The battery produces an electric field in space. Does the capacitor become charged?



QC4 Optics. In order to see correctly, humans swimming underwater need goggles, but fish do not. Explain why.

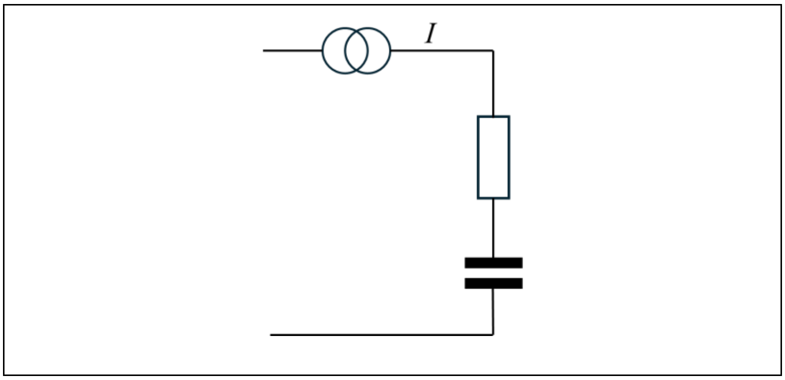
QC5 Hydrogen atom model. A proposed model of the hydrogen atom has the proton surrounded by a diffuse shell which contains the electronic charge. Will this model work?



QC6 Fields. Dust particles form a homogeneous spherical cloud. Each particle has mass *m* and bears the electronic charge *e*.

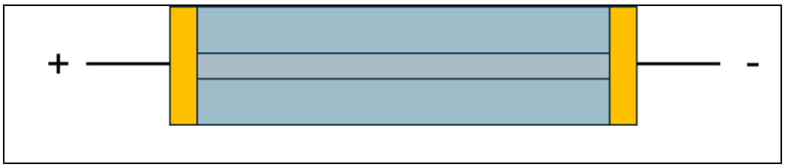
Assuming the cloud is stable, derive an expression for the particles' mass in terms of their charge.

QC8. DC Circuits. A 'current source' sends a *constant current I* through a resistor and capacitor in series. Sketch graphs of the voltage across the resistor and the capacitor, as a function of time, from the moment the source is switched on.



QC9 Wind Turbines. Explain why it is impossible for a wind turbine to extract all the power from the incoming wind.

QC10. Current through a mercury column. When current is passed through a column of mercury at the bottom of a tube, there is a visible deformation of the column. Explain the nature and cause of this deformation.



QC11. Pulses on a long spring. An observer takes a photo of a long spring which shows no transverse displacements along its entire length. A photo taken a few seconds later shows two transverse pulses. Where did they come from?

QC12. Energy conservation. An elementary school child jumps on the pressure pad of a 'stomp rocket'. Estimate the maximum height the rocket can reach.

QC13. Current in a wire. When a voltage V is applied to a conductor of length d then a current will flow due to charge carriers moving through the body of the conductor. But the charge carriers move because of the electric field $E = V/d$ across the wire. But electrical fields are produced by electrical charge. Where is this charge?

QC14. Balancing a broomstick. An inverted broomstick rests in your hand, and you are free to move your hand in any which way you wish. How would you move your hand to ensure the broomstick remains vertical for all times?

QC15. Energy and Power. Estimate, in Joules, the work done in building the Great Pyramid, a tetrahedron of approximate base length 200m and height 150m from limestone blocks of density 2000 kg/m^3. If you could use the power generated by a wind farm comprising 8 x 1.5MW turbines, how long would the job take?

QC16 Energy and Power. A river flows over a weir. The flow rate is $5m^3/s$ and the difference in water level above and below the weir is 2m. Estimate how many homes could be powered if the water power could be converted to electricity.

QC17 Pressure. A bottle containing a fizzy drink is shaken and opened carefully so that all the gas escapes and no liquid is lost. Is there any change in the hydrostatic pressure at the bottom of the bottle?

QC18 Optics. The magnification produced by a telescope depends only on the focal length of the lenses used. There is therefore little point in increasing the aperture of the telescope.

QC19 Current and Charge. Two wires carrying currents in the same direction attract each other, but electrons repel each other, therefore currents in wires cannot be due to the motion of electrons. Explain the shortcoming(s) of this reasoning.

QC20 Centripetal and Centrifugal Force. A mass at the end of a string rotates in the horizontal plane at constant speed. The centripetal force needed is provided by the string, and according to Newton's 3^{rd} law there is an equal and opposite centrifugal force acting outwards. Therefore there is no total force on the mass. Explain the shortcoming(s) of this reasoning.