Comp3402   Consistency and Validation

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Purpose	(i) To learn how to test a collection of languages sentences for consistency using Boolean logic, (ii) To prove or disprove the validity of an argument, presented in language, using Boolean logic, (iii) To appreciate how language and logic are closely connected
Files Required	Logisim Software (open source)
ILO Contribution	LO 4
Send to Me	nix
Homework	Read chapter 11 (WIP)

1   Preparation: Identification of atomic clauses

For each of the following sentences or sentence combinations identify the constituent atomic clauses

1	Sentence (a)	**Either the Vicar or the Butler shot the Earl**

2	Sentence (a)	**The gunmen are armed, but the hostages are not in danger**

3	Sentence (a)	**Holmes was on the case and Watson was on the case**
	Sentence (b)	**Watson was not on the case**

4	Sentence (a)	**If Holmes solved the crime, then Lestrade took the criminal**
	Sentence (b)	**If the criminal escaped, then Lestrade did not take him**

5	Sentence (a)	**If she's made of wood then she's a witch**
	Sentence (b)	**If she weighs the same as a duck then she's made of wood**
	Sentence (c)	**If she weighs the same as a duck then she's made of wood**
	Sentence (d)	**Therefore, she's a witch.**

2   Consistency: Holmes and Watson were on the case.

Consider the two sentences

> (1) **Holmes and Watson were on the case.**
> (2) **Either Holmes was not on the case, or Watson was**

(a) Extract the two atomic clauses

> **A =**
> **B =**

(b) Now transcribe the starting two sentences using the atomic clauses and Boolean algebra

> (1)
> (2)

(c) Now use these Boolean terms to form an expression **L = …**   when both sentence (1) **_and_** sentence (2) are true.

> **L =**

(d) Now complete the truth table like this:

(i) Put the Boolean expressions from (b) into columns 3 and 4
(ii) Put a **1** into column 5 where columns 3 and 4 both contain **1**s
(iii) In the sixth column write down the mini-terms corresponding to each row where there is a **1.**
(iv) In the seventh column, write down in simple English the meaning of any mini-term.

A	**B**	term (1)	term (2)	**1** if both true	mini-term	Simple English
0	0					
0	1					
1	0					
1	1					

(e) Are there any consistent solutions to the sentences given?
(f) If you like, build a digital circuit for the expression **L = …** and make sure it agrees with your truth table

3   Consistency: Too Many Detectives

Working as above, look for any consistent solutions to the sentences below. You will first need to find 3 atomic clauses.

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(1) **If Holmes solved the crime, then Lestrade took the criminal.**
(2) **If the criminal escaped, then Lestrade did not take the criminal.**
(3) On this day **Holmes solved the crime, but the criminal escaped.**

So, what happened on this day?

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4   Consistency: Smiley's People

Working as above, you will find one consistent solution. You will first need to find 3 atomic clauses.

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(1) **Smiley is an English spy.**
(2) **Smiley is not both a Russian and an English spy.**
(3) **If Smiley is a Cad then he is a Russian spy.**

Describe Smiley.

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4   Argument Validity: Modus Tollens

Take the atomic sentences

A = There is a fault
B = It will blow up

Let's test the validity of the following argument

If there is a fault it will blow up. It will not blow up, therefore there is no fault.

There are two premises and one conclusion.

(a) Write down Boolean expressions for the two premises, and write down the ***negated*** expression for the conclusion.

Premise 1
Premise 2
Negated Conclusion

(b) Now complete the truth table putting expressions for premises and negated conclusions in additional columns. In the column labelled **1** , for each row, write a **1** if Prem1, Prem2 and ~Conc are all **1**.

A	B	Prem1	Prem2	~Conc	1
0	0				
0	1				
1	0				
1	1				

(c) If the argument is true, then the last column should have all **0**. If there is a **1** then this is a counter example. In this case write down the mini-term and write this counter example in simple English.

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5 Argument Validity: Consequentia Mirabilis

Consider this argument

If there is proof, then there is proof.
If there is no proof, then there is proof.
Therefore, there is proof.

Prove that this argument is true

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6 Argument Validity: Fallacy of denying the Antecedent

Consider this argument

If there is a fault, then it will blow up.
There is no fault.
Therefore, it will not blow up.

Prove that this argument is false and explain the counter example you will discover.

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7 Roundup – For the Position Paper

The material here is a great demonstration of how logic and language are related. It demonstrates the power of logic in showing if a series of human utterances or written text hang together and makes sense (Consistency).

Logic is also able to show whether or not an argument is valid or not. Imagine a courtroom trial where prosecution and defence make their arguments as inputs to a program which then outputs the verdict! This was the goal of the German mathematician Leibnitz (1646 – 1716).