Chapter 5
Wind Farms

5.1 A brief Introduction

The key to understanding how to design successful wind farms centres on understanding the wake produced behind a single turbine, and how this interacts with downwind turbines. The complexity of the wake structure over a complete wind farm is shown in the photo below for the 'Horns Rev II' farm.

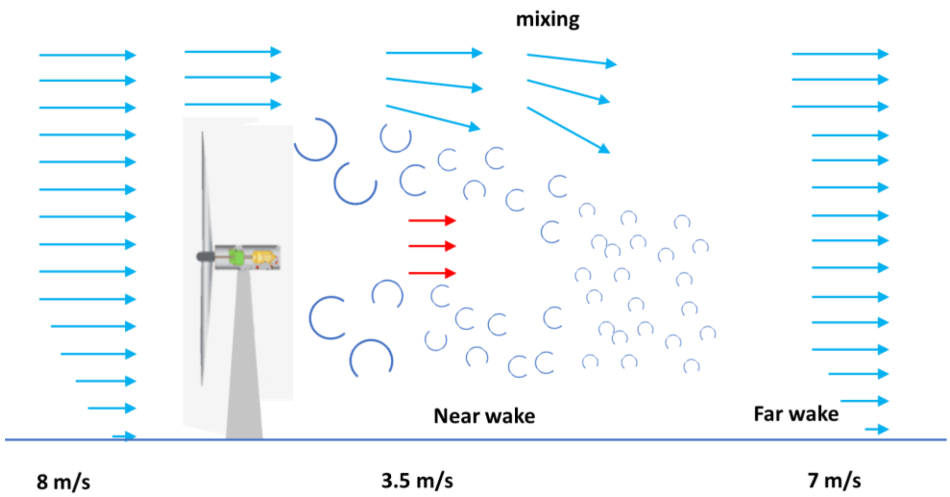
Photo by: Bel Air Aviation Denmark – Helicopter Services. January 26th, 2016.

You can see a wake of vortices produced by each turbine and that some downwind turbines lay in the wake of upwind turbines. As we shall see this air turbulence has both positive and negative effects. It allows *mixing* of the slower air immediately behind the turbine rotor with surrounding faster air. This allows the wind to 'recover' speed in the wake at large distances from the turbine. So, while a turbine downstream does not experience the full windspeed, it experiences a speed quite close to it. So that's a good effect of turbulence. A bad effect is that turbulence can cause large mechanical loads on downwind turbines as the quasi-periodic vortices impact on downwind rotors. Now let's consider this mixing effect.

5.2 The Single Wake

Mixing of Air

To understand how the wind speed varies along a single wake, we need to see how wind is mixed. This is shown in the diagram below.



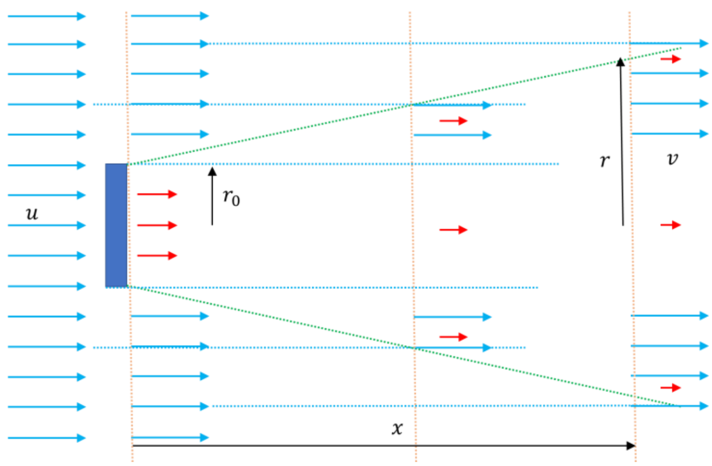
The wind enters from the left (8 m/s), and immediately behind the turbine blade a system of vortices is created. This is a region of low wind speed (3 m/s, red arrows). So, the wind speed drops dramatically as the wind traverses the turbine rotor. Then the shed vortices travel downstream, and their circulation starts to dissipate. Also, wind (at full speed) which has not crossed the turbine starts to mix in with the vortices and so the wind speed in the wake starts to increase.

There are two regions to the wake; in the *near wake* the wind speed is low (since the turbine has extracted energy). This region occupies several rotor diameters. In the *far wake* the mixing is complete and the wind velocity has increased (7 m/s – blue arrows). This gives some hints at where not to place a turbine downwind; the near wake region must be avoided.

The Jensen Model

This is the earliest (1983) and simplest model of wake expansion but is still used in software today (including Floris). Its purpose was to increase the efficiency of wind farms by locating turbines to avoid wake losses.

The model assumes that the wake diameter starts with the value of the turbine diameter and expands linearly with distance from the turbine. It completely neglects the near wake, so is based on a progressive mixing of the slow wind behind the turbine disk with the fast wind outside this disc. This is shown in the diagram below



The blue arrows show the wind starting with speed u on the left. The reduced wind speed after the turbine u_R is shown by the shorter red arrows, as the wake spreads linearly these are diluted as faster wind mixes in from outside the cylinder around the turbine. The speed at the right is v.

The expression for the velocity at the right (derived in the appendix) is

$$v = u\left[1 - 2a\left(\frac{r_0}{r_0 + \alpha x}\right)^2\right] \qquad (1)$$

with

$$a = \frac{1}{2}\left(1 - \frac{u_R}{u}\right) \qquad (2)$$

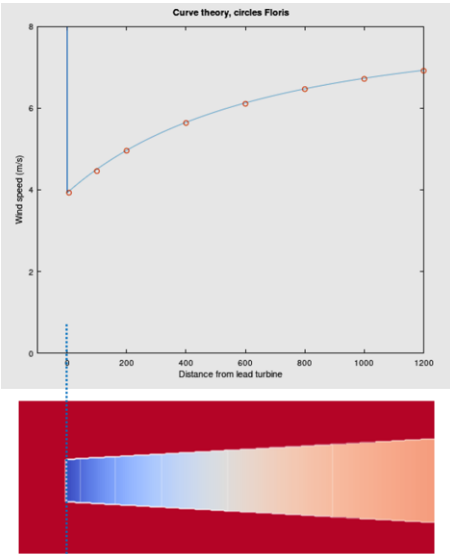


Figure 5.1 Top: plot of expression (1). Bottom: corresponding field from Floris

where u_R is the wind speed immediately after the rotor (to its right). This constant is known as the *induction factor*. For a free wind speed of 8 m/s and a typical $u_R = 4$ m/s the induction factor is 0.25. It's just an indication of the effect of the turbine on the wind velocity near the turbine.

The parameter α determines how quickly the wave expands with distance from the turbine, whether the cone is more or less pointed. This is an empirical parameter which is about 0.075 for on-shore or 0.04 for off-shore. This means that on-shore wakes tend to spread wider.

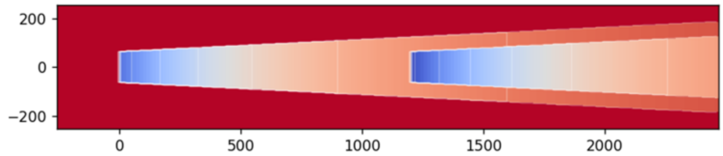
Figure 5.1 shows a plot of expression (1). The drop across the rotor is shown (not part of the expression) the speed dropping from 8 m/s to around 4m/s. Moving down the wake you can see how the speed is rising slowly ending up around 7 m/s at a distance 1200 metres (1.2 km) from the turbine.

The bottom image shows the output from Floris; here red is 8 m/s and blue around 4 m/s. This shows how the Jensen wake develops in 2D, and you can see the wind speed increasing in the wake.

5.3 Interacting Wakes

Two Turbines with Lateral Displacement

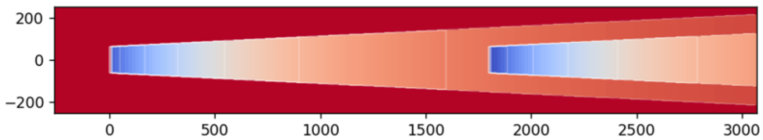
This is the starting point to understand wind turbine placement in farms, where a second turbine is place wholly in the wake of the lead turbine. Two scenarios are shown in the figure below. The first shows the second turbine placed at a separation of 1.2 km. The powers produced by each



Turbine 0 power = 1.70e+06 (= 1695368.6)
Turbine 1 power = 1.10e+06 (= 1098534.8)

Total Power = 2.79e+06 (= 2793903.5)

turbine are shown. The second turbine is producing less power since it is receiving wind at a lower speed; Floris tells us this is 6.92 m/s. That’s a consequence of being in the wake. If we increase the separation, then we shall increase the power generated by the second turbine shown below for a separation of 1.8 km. This is a consequence of more free wind mixing in.



Turbine 0 power = 1.70e+06 (= 1695368.6)
Turbine 1 power = 1.29e+06 (= 1293864.8)

Total Power = 2.99e+06 (= 2989233.4)

Here the wind in front of the turbine is now 7.3 m/s due to the wake expansion and free wind at 8 m/s mixing in, as discussed above.

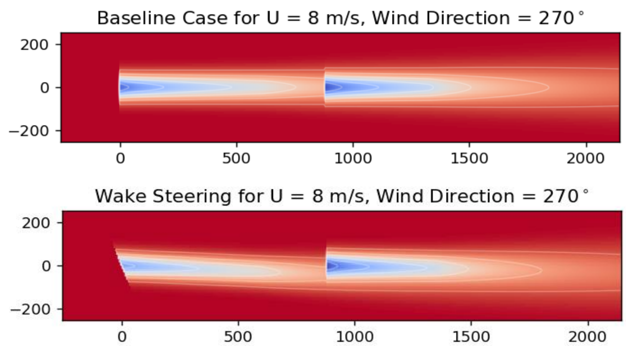
> We can do a nice little calculation at this point. Since we know that power is proportional to wind velocity *cubed* then the ratios calculated here should be the same
>
> $$\left(\frac{7.3}{6.92}\right)^3 = 1.174 \qquad \frac{1.29}{1.10} = 1.173$$

Two Turbines with Wake Steering

Wake steering is a promising area of wind farm research. The idea is to angle the lead turbine by changing its yaw so that its wake is partially deflected from the downwind turbine. The idea is shown in the Floris image below

The baseline case without wake steering produces a total power of 4.55 MW, and with wake (yaw = 20 degs)steering this is increased to 4.83 MW, just over 6%. This is a huge increase which is obtained for no additional capital cost.

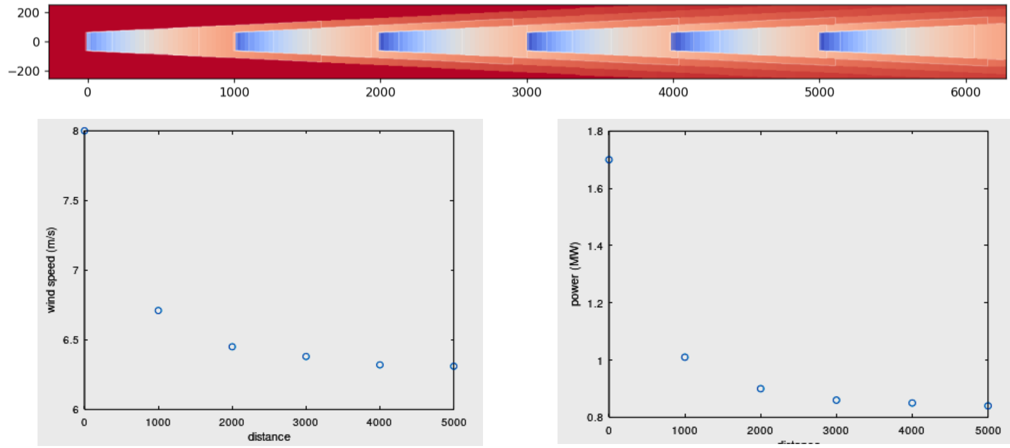
How wake steering works is relatively straightforward to understand. Due to the deflection of the lead turbine wake, the second turbine is receiving wind from outside the wake.



The average speed received by the second turbine has increased, you can clearly see this in the above figure. It is interesting to see that the wake is steered in the opposite direction to the yaw angle. This is a consequence of resolving the turbine thrust into directions parallel and orthogonal to the wind direction. Material for an appendix.

A Column of Several Turbines

If you search for wind farm layouts, you will find that many are composed of repeated columns of turbines. Therefore, it makes sense to study a column as shown in the graphic below.



The Floris field is shown together with the windspeeds and individual turbine powers at each location. The results are interesting. Of course, we expect turbines further downwind to produce less power, and they do! But the decrease in wind speed and power slows down after the first two or three turbines, and both tend to asymptotic values. The wind speed tends to just under 6.5 m/s and the power to just above 0.8 MW. So, this suggests that it is very reasonable to build long columns of turbines.

Changing the Layout

One approach to designing a 2D windfarm is to start with a 1D scenario, and to 'morph' this into 2D. For example, we could take the 6 turbine column and displace every other turbine sideways to create two columns of 3 turbines. We could calculate how the power changes as a function of how much we displace the turbines.

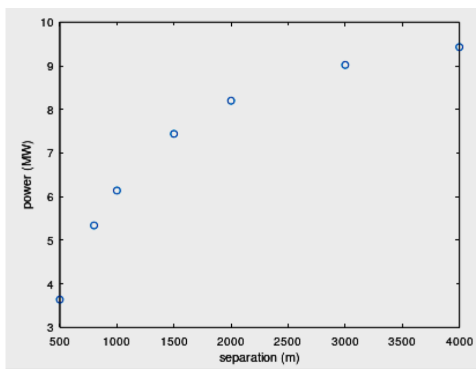


Figure 5.2 Power produced by a column of 6 turbines as a function of their separation.

But first we need to understand the effect of changing the inter-turbine separation on the total power generated. This can be done by running several Floris computations. The results of such a calculation are shown in Fig. 5.2. As expected, the power increases with separation, since more free wind speed is available to all turbines as it mixes into the wakes. But the rate of improvement slows down, and, once more, we seem to be heading for an asymptote. Of course, increasing separation comes with a cost; the cost of the cables

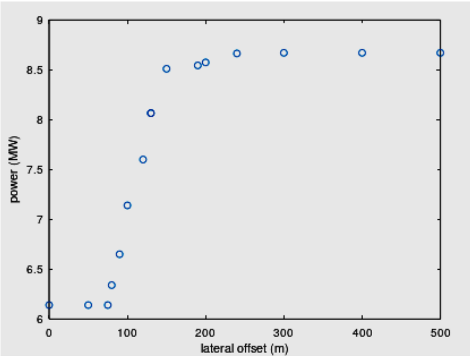


Figure 5.3 Power as a function of offset between two columns of three turbine starting from 0 (all turbines in a row)

connecting the turbines to the electricity sub-station. This sounds like the beginning of an optimization problem.

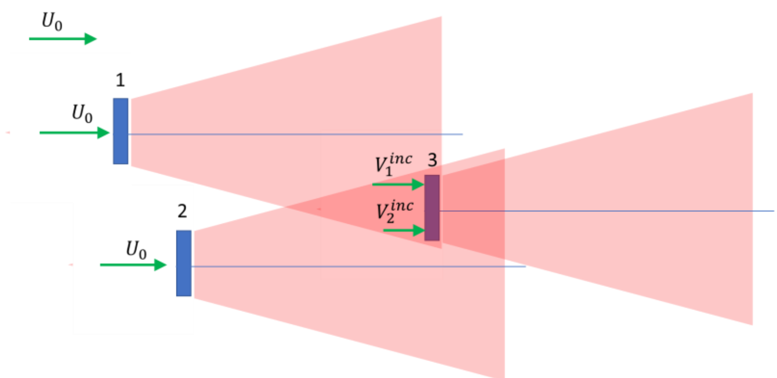
Turning now to morphing the 6 turbine column into two adjacent columns of 3 turbines, we find a very interesting result as the separation between the columns ('offset') is increased. This is shown in Fig. 5.3. As we start to increase the offset from zero there is initially no increase in power until the separation is between 80 and 90 m. Then there is a sudden increase in power with separation until we reach around 150m or so, after which the power remains pretty constant.

Such a plot is indicative of a *binary* situation, where we change from one behaviour or arrangement to another. You may have seen such a plot in the study of binary electronic circuits. I feel sure you can work out what is going on here, it would be a shame to give you the answer just now.

Wake Superposition

We need to understand how to calculate the wind velocity incident on a turbine, where the turbine lies in the wakes of more than one upwind turbine.

Consider the situation shown below. There are two upwind



turbines 1 and 2, and a downwind turbine 3 which lies in the wakes of both 1 and 2. The ambient wind speed is U_0 and both turbines 1 and 2 experience this maximal speed. The question is how do we find the speed experienced by turbine 3? We know how to calculate V_1^{inc}, the wind speed downwind in the wake of turbine 1, at the location of turbine 2. To do this we use equation (1) where x is the distance between turbines 1 and 3 and we set $u = U_0$ in equation (1). Likewise we can calculate V_2^{inc} the windspeed incident on turbine 3 due to the wake of turbine 1. The question now is, how do we combine these two incident speeds?

Several approaches have been proposed by wind farm *engineers*; these are based on empirical measurements and not the laws of physics. The simplest approach goes like this

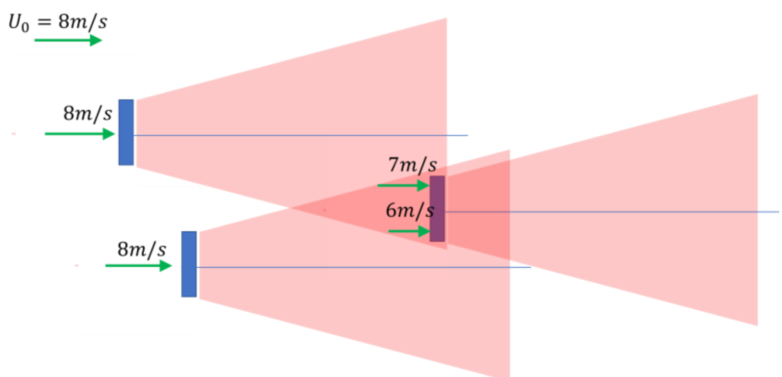
$$V_3 = U_0 - \left[\left(U_0 - V_1^{inc}\right) + \left(U_0 - V_2^{inc}\right)\right] \qquad (2)$$

Here we subtract the incident speeds $V_1^{inc}$and V_2^{inc} both from the ambient wind speed and then subtract that result from the ambient wind speed. It's interesting to re-write (2) like this

$$(U_0 - V_3) = \left(U_0 - V_1^{inc}\right) + \left(U_0 - V_2^{inc}\right) \qquad (3)$$

Here the quantities in brackets are called *velocity deficits* how much the wind speed at a turbine differs from the ambient wind speed. The approach described by equation (3) is simply stated, *the velocity deficit at the downwind turbine is the sum of the velocity deficits of the upwind turbines.*

To see this in action, let's take a toy problem shown below where the incident speeds have been calculated as 7 m/s from turbine 1 and 6 m/s from turbine 2.



Plugging these numbers into equation (3) together with the ambient windspeed of 8 m/s we find

$$(8 - V_3) = (8 - 7) + (8 - 6)$$

which solves for

$$V_3 = 5\ m/s$$

In general, if our turbine j lies in the wakes of N upwind turbines, then we have

$$\left(U_0 - V_j\right) = \sum_{i=1}^{N} (U_0 - V_i) \qquad (4)$$

This summing of velocity deficits gives reasonably good agreement with experiments, but not the best. Better results are obtained by summing the deficits squared like this

$$\left(U_0 - V_j\right)^2 = \sum_{i=1}^{N} (U_0 - V_i)^2 \qquad (5)$$

and this is how Floris superposes its wakes.

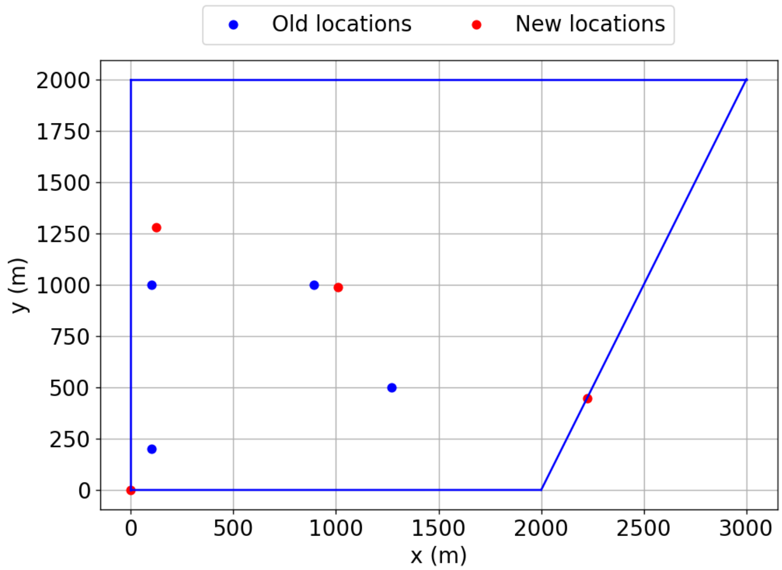
5.4 Layout Optimization

This is a very complex topic, there are many algorithms available. Here we shall restrict the discussion to the technique available in Floris.

The whole idea centres on ‘constrained optimization. Let’s take these two words separately. Optimization means changing the position of the turbines in a farm to achieve the maximum Annual Energy Production (AEP). Constrained means we have some constraints on the problem, for example the turbines must be located within a given boundary, and there may be some places where turbines may not be located, e.g., in a school playground. So, optimization and constraints need to be taken together.

Floris provides us with a range of problem solvers, e.g., the AEP given frequencies of wind speeds and direction subject to the constraints of a given boundary and minimum distance between the turbines. In addition, constraints on the turbine heights may be supplied, and optimization may be made on the Cost of Energy (COE) rather than the AEP.

The example shown below was computed using Floris, to optimize the turbine layout to maximize the AEP. Wind speed and direction were provided, and boundary constraints asserted.



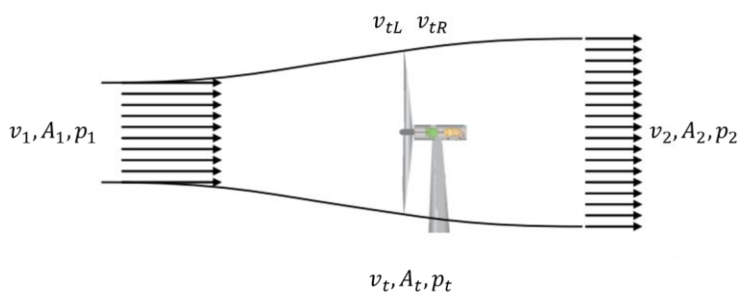
The total gain in AEP due to the optimization was 3.0%.

The actual optimization routine used the ‘sequential quadratic programming’ algorithm; the maths behind this is really beyond this course.

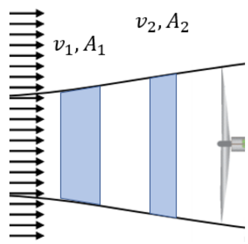
5.5 Appendix Material

Derivation of Power and Torque and Betz Limit

Let's have a look at wind passing a single turbine. The diagram below shows air passing through a stream-tube, all the air remains in that tube and we are not yet considering mixing.



The parameters are air velocity, pressure and area of the stream tube upwind, at the turbine rotor and downwind. Arrows show that the wind speed is decreasing as can be understood from the diagram below



The blue areas show a given element of volume as it moves through the tube. Assuming air is incompressible then the volume stays the same. Therefore, the length of the element must get smaller since the area gets larger. So, we have for the volumes

$$\Delta l_1 A_1 = \Delta l_2 A_2 \qquad (A1)$$

But the above snapshot shows movement over the same time interval so dividing the above by this interval we find how the volume changes with time

$$\frac{\Delta l_1}{\Delta t} A_1 = \frac{\Delta l_2}{\Delta t} A_2 = v_1 A_1 = v_2 A_2 \qquad (A2)$$

So we see that the air slows down. Multiplying by the air density ρ we get an expression for the rate of change of mass (the same at all places)

$$\frac{\Delta m}{\Delta t} = \rho A_1 v_1 = \rho A_t v_t = \rho A_2 v_2 \qquad (A3)$$

Hence the rate of change in momentum of the air stream between when it enters and when it leaves the stream-tube is

$$\frac{\Delta m}{\Delta t} v_1 - \frac{\Delta m}{\Delta t} v_2 = \rho A_t v_t (v_1 - v_2) \qquad (A4)$$

and this is caused by the thrust on the turbine disc. We can use Bernoulli on in the stream-tube at the left and right sides of the turbine disc

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_{tL} + \tfrac{1}{2}\rho v_{tL}^2 \qquad (A5a)$$

$$p_{tR} + \tfrac{1}{2}\rho v_{tR}^2 = p_2 + \tfrac{1}{2}\rho v_2^2 \qquad (A5b)$$

Assuming that $p_1 = p_2$ and $V_{tL} = V_{tR}$ then the pressure difference across the disc is

$$p_{tL} - p_{tR} = \tfrac{1}{2}\rho (v_1^2 - v_2^2) \qquad (A6)$$

Since force is pressure times area, we can use this to get another expression for force on the disc

$$F = \tfrac{1}{2}\rho (v_1^2 - v_2^2) A_t \qquad (A7)$$

Equating the two expressions for force we find

$$\rho A_t v_t (v_1 - v_2) = \frac{1}{2} \rho (v_1^2 - v_2^2) A_t$$

$$v_t = \frac{1}{2} (v_1 + v_2) \qquad (A8)$$

which tells us the windspeed at the turbine is the average of the upwind and downwind wind speeds.

Finally, we have an expression for the power delivered to the turbine. Power is force x velocity, so at the turbine we find

$$P = F_t v_t = 2\rho A_t v_t^2 (v_1 - v_t) \qquad (A9)$$

This is an interesting expression which shows how the power depends on the wind speed near the turbine. There are two factors, v_t^3which increases with v_t and $(v_1 - v_t)$ which decreases with v_t, so we expect the power - $v_t$curve to have a peak which it does. To find the peak we proceed as usual,

$$\frac{\partial P}{\partial v_t} = 0,$$

which gives the result $v_t = \frac{2}{3} v_1$ and also $v_2 = \frac{1}{3} v_1$.

Finally we turn to the maximum power expression, plugging this value of v_t into the expression for power P we find

$$P_{max} = \frac{8}{27} \rho A_t v_1^3 \qquad (A10)$$

and if we define the power coefficient as

$$C_p = \frac{P}{\frac{1}{2} \rho A_t v_1^3}$$

then we find

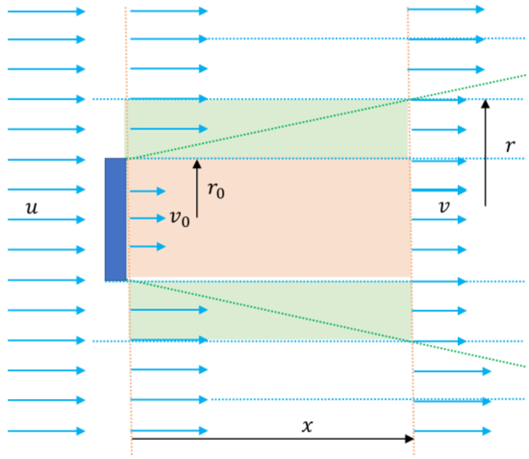
$$C_{p,max} = \frac{16}{27} = 0.593 \qquad (A11)$$

This is the maximum achievable efficiency of a wind turbine (59.3%) and is known as the Betz limit

Derivation of the Jensen Wake Expression

Here we shall derive equation (1) the wind speed variation inside a wake taking Jensen’s approach assuming we are in the far-field wake where perfect mixing is occurring.

Here's a diagram which highlights the regions of air arriving at the plane on the right, a distance *x* from the turbine rotor



The total air arriving is the sum of the air from the red and green regions. The red is the slow air (speed just behind the rotor) and the green is the fast air (free). The respective rates of arrival of air masses are

$$\rho\pi r_o^2 v_0 \quad (red)$$

$$\rho\pi(r^2 - r_o^2)u \quad (green)$$

So we have

$$\rho\pi r_o^2 v_0 + \rho\pi(r^2 - r_o^2)u = \rho\pi r^2 v \qquad (A12)$$

hence

$$r_o^2 v_0 + (r^2 - r_o^2)u = r^2 v \qquad (A13)$$

Now we define

$$a = \frac{u - v_0}{u}$$

so that $v_0 = u(1 - a)$ and using (A8) we find

$$v = u(1 - 2a) \qquad (A14)$$

This is correct for a situation with a near and a far wake, but Jensen ignored the near wake and used (A14) as an expression for v_0.

Substituting (A14) into (A13) leads to expression (1).