Chapter 4
Wind Turbine Technology

4.1 A brief Introduction

If you take a look at the 'Live Status' of the UK National Grid at the link here,[1] you will find some interesting facts about our energy demand and generation at this moment in time. Almost half of our current production comes from gas, a respectable third from nuclear and around one sixth from wind. This division may reflect our history of energy technologies (from which we cannot escape). Renewables are currently being highlighted in the media and academic journal publications.

We shall explore two technologies: Wind Energy and Vibration Energy Harvesting (VEH). The former is now well established, you know what this is, you have seen wind turbines. There is still much research to be done, to increase their efficiency of energy production. This means increasing their physical size, which requires understanding of the tower and blade mechanics; as their designs get larger, the blade and tower structures start to flex.

Fig.4.1 is a graphic from NREL[2]s recent newsletter showing the evolution of wind turbine sizes and heights. You may like to consider where a football pitch lies on this scale. Wind energy is the fastest growing energy solution in the world, with an associated demand in engineers and researchers in the field.

---

[1] https://grid.iamkate.com/
[2] https://www.nrel.gov/wind/ National Renewable Energy Laboratory. Part of the U.S. Department of Energy.

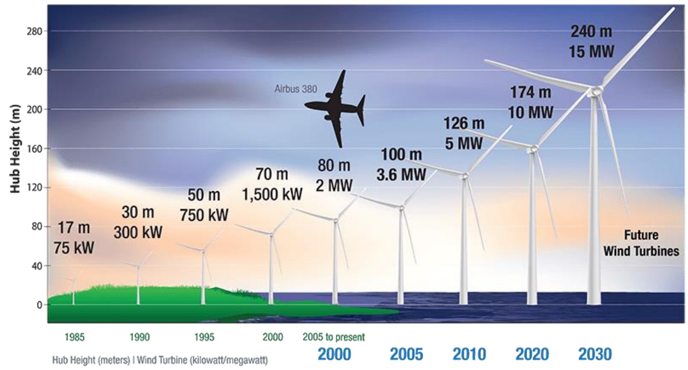


Figure 4.1 Evolution of wind turbine sizes, heights and power production. Illustration by Josh Bauer, NREL.

This chapter is about wind turbines. We shall explore three aspects of wind energy. First how a single turbine works to extract energy from the wind optimally. This is no simple task since wind-speeds do vary quite a lot. Second, we shall look at the turbulence created in the wake of a wind turbine and the effect this has on downstream turbines. Finally, we shall consider the design of Wind Farms; how the placement of each turbine within a farm can be decided to optimize something (like energy produced). All references will be to actual turbines and farms, both commercial and research.

VEHs are very new, in a research stage; they are destined to capture energy from ‘things that move’. Think of a shock absorber component of your car suspension; this damps vibration by extracting it and dissipating it as heat. Bad. So, let’s replace the shock absorber with a small generator, to capture the motion and charge a battery. Good. Where else could we do this? Put a VEH into our shoes, or under a train track; when a train rumbles along it depresses track segments, so let’s convert this depression to electricity. VEHs will be the subject of a separate chapter.

4.2 A Philosophical Aside

Before we dive into the topic, let's just consider the *notion* of 'Renewable Energy' which may be loosely defined as 'energy that is replenished on a human timescale' (aka your lifetime), and takes on the form of sunlight, wind, hydroelectric, geothermal and others. The implication is that it will be a permanent resource, it is somehow renewed or replaced, and over time it will not diminish. Of course, that is simply nonsense, but there is something more worrying than simple nonsense.

Think about the wind turbines (renewable energy). They remove energy from the local wind system which is part of the global wind system which is a part of our climate. Change the Jet Stream and reap the consequences. Therefore, wind turbines directly affect our climate, they cause climate change, just like burning fossil fuels, but someone forgot to model and simulate this change.

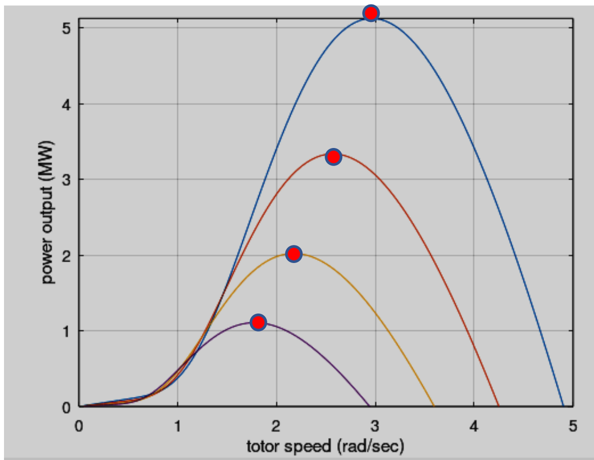
4.3 Wind Energy Modelling and Simulation

In this chapter we shall explore wind energy through a number of lenses. First, we shall look at a single wind turbine and how it is controlled to maximize energy produced as wind speed changes. Second, we shall consider the fluid flow around the turbine, how the fluid downstream from the turbine is turbulent, and may influence downstream turbines. Third, we shall look at wind farms and how turbine placement can optimize energy production, based on our above results. But I told you that already.

4.4 Wind Turbine Basics

Here we take an overview of the structure and operation of a wind turbine. Several mathematical expressions will be presented; these are the ones we need to understand how to code a sophisticated simulation. The explanations of these expressions will be presented at the end of this chapter for the interested reader. First, we need to know some facts about how a wind turbine absorbs power from the wind. Obviously, the wind causes the turbine to rotate, we use the symbol ω to represent the turbine's *angular velocity*. This is measured in radians per second, where one revolution per second would be 2π radians per second which is just over 6 rad/sec.

The wind has a certain velocity *v(t)* which depends on time (hence the brackets showing it is a function of time), and so the angular velocity will change, when the wind speed is high, then the angular velocity will be large. So, the wind speed and angular velocity are related. The question is how. We can find the answer by looking at what effects the power produced by a real wind turbine. This is shown in the diagram below.



Up the side we have power output in Mega-Watts and along the bottom is the rotor speed in rad/sec. Each curve is for a particular wind speed, for 9 m/s, 11 m/s, 13 m/s and 15 m/s. We can see, for each wind speed there is a maximum of the curve where the turbine is outputting maximum power for that wind speed. Each maximum occurs for a particular rotor speed, so we want to adjust the rotor speed to match the maximum of the power curve at each wind speed.

The job of the turbine controller is therefore to adjust the rotation speed to match the wind speed.

Now something interesting happens if you look at the curve maxima (shown as red dots). If you divide the rotor speed by the associated wind speed, then something interesting happens. Let's see.

$\boldsymbol{\omega}$	2.96	2.55	2.15	1.77
$\boldsymbol{v}$	15	13	11	9
$\boldsymbol{\omega/v}$	0.197	0.196	0.196	0.197

This ratio is more or less constant for this particular turbine, and therefore it is significant. It means that the turbine controller must adjust ω to the wind speed, so that this ratio is obtained.

Wind turbine theory uses this result but changes how the above ratio is calculated, this is called the 'tip speed ratio'. There is another important velocity, that of the tip of the turbine blades as they rotate. If the blades have radius *R* and rotate with angular velocity ω radians per second, then the tip has a speed in metres per second given by

$$v = R\omega \qquad (1)$$

Then we define the 'tip speed ratio', which is the ratio of the tip speed to the wind speed,

$$\lambda = \frac{R\omega}{v} \qquad (2)$$

It turns out that this quantity has an optimal value (not too small, not too large) when the turbine extracts the maximum power for any given wind speed *v*. This ratio is about 4, but depends on the particular turbine.

So, to recap, the turbine controller adjusts it rotation velocity ω to the wind speed so that equation (2) is satisfied. Then the turbine extracts the maximum power from the wind (the red dots on the above diagram)

Now, the power available in wind with velocity *v* turns out to be

$$P_{wind} = \frac{1}{2}\rho A v^3 \qquad (3)$$

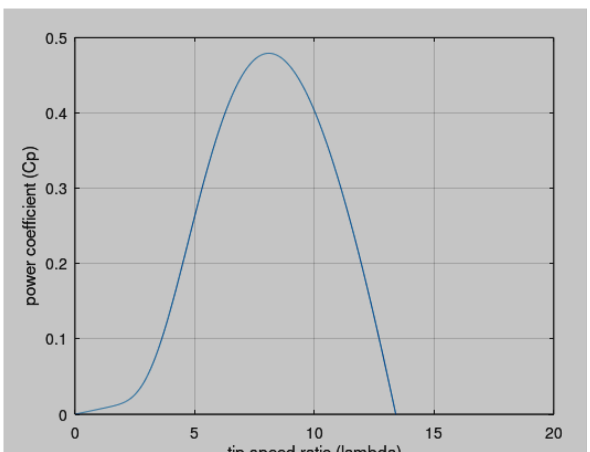
where A is the area swept out by the rotor blades, and ρ is the density of air at the hight where the turbine is located. This expression makes sense. However, a turbine can never extract this amount of power. If it did, then the air would come to a stop after passing through the turbine. This cannot be true, since it would prevent any more air from passing through the turbine. As we shall see, the maximum amount of power is around 59% (0.59) of the above power; this is the 'Betz' limit. But real turbines can only approach this theoretical limit, and how much they actually extract is dependent on the blade design.

Each turbine is characterized by a *power coefficient* C_p which is the ratio of power captured by the turbine to the power in the wind,

$$C_p = \frac{P_{extracted}}{P_{wind}} \qquad (4)$$

This coefficient will be less than 0.59 which is the Betz limit. Typical values for modern wind turbines are in the range of 0.47-0.50 which means they extract about 50% of the available wind energy. So, there is still some wind left over to blow smoke away from fossil-fuel fires. It turns out that the power coefficient C_p depends on the tip-speed ratio λ and also on the *blade pitch* β. So, you will see this expressed as $C_p(\lambda, \beta)$.

Let's look at how C_p depends on tip-speed ratio λ for pitch $\beta = 0$, (we'll look at the *blade pitch* β dependence later). Here's a plot made from an analytical solution. You can see that the peak of C_p at 0.479 is close to the Betz limit, and this occurs for $\lambda = 8$.



For the Seimens SWT-3.6-107 3.6MW turbines, installed on the Walney wind farm, this means a power generated of (0.479)(3.6) = 1.72MW, with the blade tip moving at 8 times the wind speed, (easy to achieve since the blade length is 54m.

Later we shall consider how to ensure that a turbine is operating optimally, at the peak of the C_p curve. But while we have the tip-speed ratio in mind, let's prepare some ground. Let's say this ratio is optimal at 8, then the wind speed drops. From equation (2) we see the ratio increases, so it is too large. To reduce it we must reduce ω, the speed of rotation of the turbine.

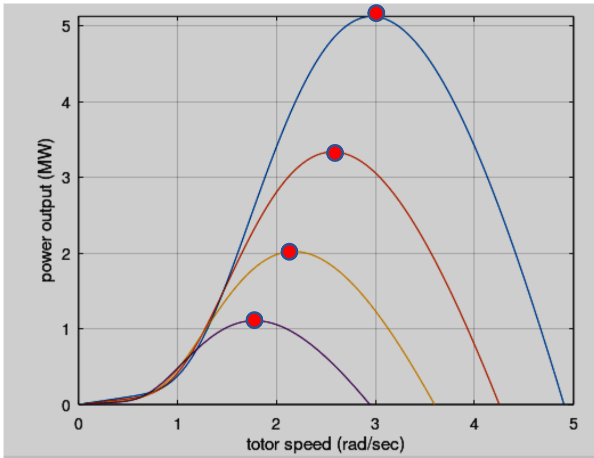
Power Output Curves

The $C_p - \lambda$ curve is fundamental in understanding the details of wind turbine control (below), but it is somewhat unintuitive since λ conflates two variables, wind speed v and turbine rotational velocity ω. Using the expressions (2), (3), (4) we can derive expressions for the power output in terms of ω or v.

$$P_{turbine} = \frac{1}{2}\rho\pi R^5 \frac{C_p(\lambda, \beta)\omega^3}{\lambda^3} \qquad (5a)$$

$$P_{turbine} = \frac{1}{2}\rho\pi R^2 C_p(\lambda, \beta) v^3 \qquad (5b)$$

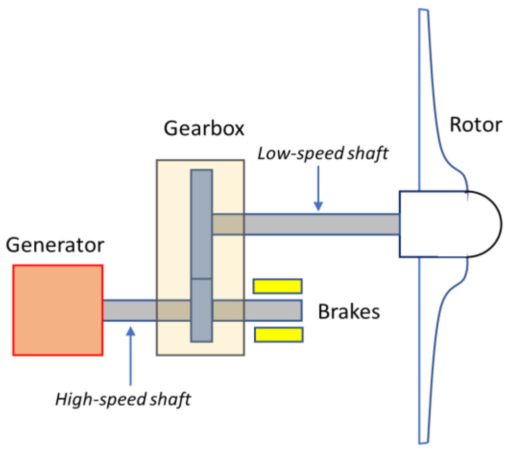
Expression (5a) lets us plot out the turbine power as a function of rotor speed, for various wind speeds. So, we have separated out both parts of λ. Curves are plotted for windspeeds of 9, 11, 13, and 15 m/s.



For each wind speed, there is a particular rotor speed where the power extracted is maximum, as shown by the red dots. This, of course, produces the optimal tip-speed ratio. Join the dots and you'll get the 'maximum power point tracking (MMPT) of the turbine; the turbine is controlled (by varying its speed of rotation) so it stays on this curve as the wind speed varies.

4.5 Structure of a Wind Turbine

You have all seen the main components of a Wind Turbine; the tower, the rotor and the nacelle (which houses a gearbox and generator).



First we need to understand why there is a gearbox. The generator is ultimately connected to the grid which supplies alternating current at a frequency of 50Hz (cycles per second). The turbine *must* emit current with this frequency, it must synchronise with the grid, otherwise the grid would pull the turbine generator into synch and to do this would apply a huge mechanical load which would lead to failure.

If the generator had one pole producing current, then at 50Hz the generator would need to turn at 3000 rpm. This is much faster than the turbine blades rotate, so there must be a gearbox to convert slow blade rotation to the higher generator speed. For the NREL CART3 turbine the gearbox ratio is 43.2.

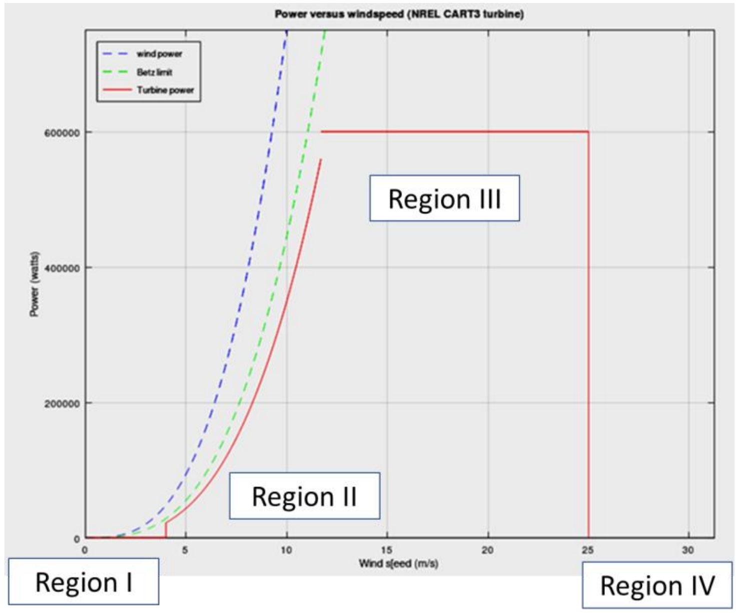
There are two main types of turbine, fixed-speed and variable speed. Fixed-speed devices are connected directly to the grid with the generator rotating at the required 3000 rpm. It turns out that if you let the turbine rotate with a variable speed, then it can get an extra 2.5% power out of the wind. Of course, it cannot be directly connected to the grid; another layer of power electronics is required to do the frequency synchronization. We shall be working with variable speed turbines

Figure 4.2 NREL CART3 Turbine. Photo by Lee Jay Fingersh NREL 54232

4.6 Regions of Operation

There are four regions of operation for a variable-speed turbine shown in the graph below for the CART3 turbine (see Fig 4.2) which we shall be modelling.

In Region 1 there is not enough power in the wind to offset turbine mechanical losses, so the turbine does not rotate. In Region 2 the turbine is operating below its rated power, here 600 kW and it speed increases with wind speed. In Region 3 the turbine is operating at its rated power, so its speed of rotation is held constant even though the wind speed may increase.



Finally in Region 4, the wind speed is too high for the turbine controller to hold it at the rated power, so the turbine shuts down.

Now we turn to discussing the turbine *controller*, systems that maintain the turbine at the desired operating point. There are different approaches to control in Region 2 and Region 3 as we shall see.

4.7 Region 2 Controller

We need to control the turbine speed of rotation; this is represented by the maths symbol ω (omega) and has units of radians per second. For a turbine rotating at one rev per second, then the angular speed is $\omega = 2\pi$ which is around 6 radians per second. The rated speed of the CART-3 turbine (in Region 3) is $\omega_{rated} = 3.952$ radians per second, so it's rotating less than once per second.

To understand how to control ω we must ask ourselves what is *causing* ω, or more specifically what is *causing ω to* *change*? Well one thing is the wind, and when this passes through the blades it exerts a *torque* on the blades causing ω the angular speed to increase. The symbol for torque is τ (tau, pronounced like cow) so if τ_{wind} is the torque on the turbine due to wind, then the rotation velocity changes like this

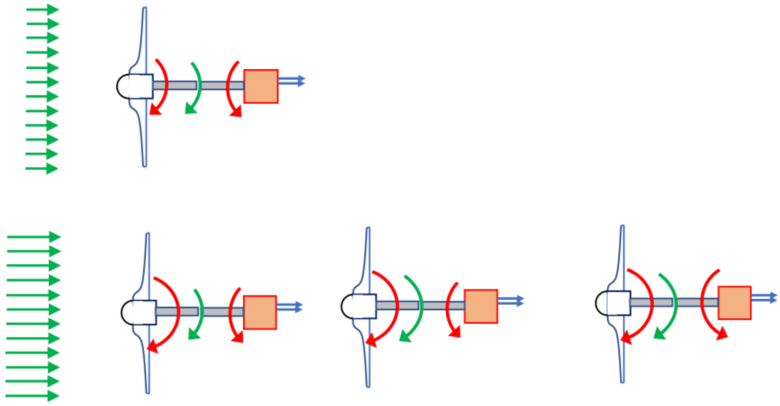
$$\frac{d\omega}{dt} = \frac{1}{J}\tau_{wind} \qquad (6)$$

The left hand side is saying omega is changing with time, and the right hand side is saying that the torque from the wind is causing this change. Also, since the right hand side is positive, then the wind torque is making the turbine speed up. The symbol J is the inertia of the turbine, playing the role of mass, more inertia means smaller rate of change of omega. So the wind makes the rotor speed up, and therefore we need something to apply torque in the opposite direction to prevent the turbine running away. The *generator* is the perfect candidate, and if τ_{gen} is the turbine generator we can complete the above expression for the change in rotation velocity

$$\frac{d\omega}{dt} = \frac{1}{J}\left(\tau_{wind} - \tau_{gen}\right) \qquad (7)$$

So, we see the generator is a crucial part of a wind turbine, as well as generating electricity, it can act as a controller to fix omega. You can see from the above expression that when the generator matches its torque to the torque from the blades, then $\tau_{wind} - \tau_{gen} = 0$ so the rate of change of omega is zero; the turbine’s speed does not change.

The diagram below should help you understand this. Top left shows a turbine in steady wind. The green arrow on the shaft indicates the rotation velocity, the left red arrow the torque on the blades, and the right arrow shows the opposing generator torque. Since these are equal there is no change in rotational velocity. The bottom line shows how the controller responds to a sudden increase in wind velocity.



The higher wind velocity increases the turbine torque which is now larger than the opposing generator torque, and so the turbine speeds up. To prevent a continual speed-up, the generator torque is increases so that it matches the turbine torque. Then the right hand side of (7) is zero, and the turbine rotates once again at a constant rotation velocity albeit higher than its initial velocity.

Now we have to add in some details. First, we introduce an expression for the torque from the wind (this will be derived in an appendix).

$$\tau_{wind} = \frac{1}{2}\rho\pi R^5 \frac{C(\lambda,\beta)}{\lambda^3}\omega^2 \qquad (8)$$

This looks formidable, but perhaps you can see the blade area πr^2 lurking there, and the only true variable on the right is omega, the rotation speed. So, this expression links aerodynamic torque and wind speed.

The simplest way to model generator-based control is to make the generator torque depend on omega in the same way, so we choose

$$\tau_{gen} = \frac{1}{2}\rho\pi R^5 K\omega^2 \qquad (9)$$

Where we need to choose a value for the constant K. Plugging (8) and (9) into (7) we find

$$\frac{d\omega}{dt} = \frac{1}{2J}\rho\pi R^5\omega^2\left(\frac{C(\lambda,\beta)}{\lambda^3} - K\right) \qquad (10)$$

Now here comes the magic; the choice of K. Remember we want the turbine to change its speed, so it is operating at the peak of the C_p curve. Then its speed does not change so $d\omega/dt = 0$. So, we choose K as follows

$$K = \frac{C_{pMax}}{\lambda_{opt}^3} \qquad (11)$$

Now expression (10) becomes

$$\frac{d\omega}{dt} = \frac{1}{2J}\rho\pi R^5\omega^2\left(\frac{C(\lambda,\beta)}{\lambda^3} - \frac{C_{pMax}}{\lambda_{opt}^3}\right) \qquad (12)$$

So, when we are operating at the peak of the $C_p$curve, we have $C_p = C_{pMax}$ and $\lambda^3 = \lambda_{opt}^3$ so the expression in the brackets is zero and the turbine speed does not change.

We can see what happens, for a given wind speed if the turbine it is turning too fast (omega too large) Since $\lambda = \omega R/v$ then we have $\lambda > \lambda_{opt}$ so, the first bracketed term in (12) is smaller than the second so the bracket is negative, so the turbine slows down. Conversely if the turbine is turning too slowly then $\lambda < \lambda_{opt}$ so the first term in (12) becomes larger than the second, the bracket is positive so the turbine speeds up.

You may feel you have lost sight of what is actually changing to effect the control. Well, this is the generator torque, expression (9) with the value of K from (11). So, the electrical load on the generator is changed depending on the rotational speed of the turbine. The electrical load determines the generator torque. Stated simply, if the turbine is rotating too fast then the generator load is increased to slow it down and *vice-versa*.

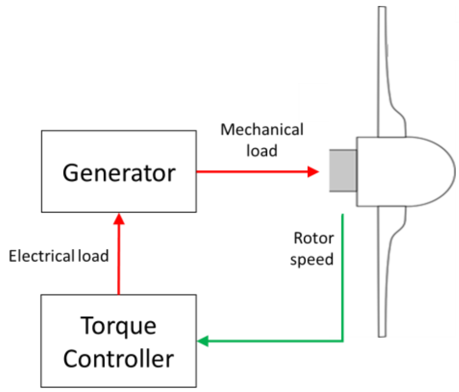


Figure 4.3 Region 2 Control Loop. Rotor speed is measured, then the controller changes the electrical load on the generator which changes the mechanical load (torque) on the turbine shaft.

For the more mathematically inclined we can explain how the controller works by looking again at expression (12) from which we derive the following inequalities,

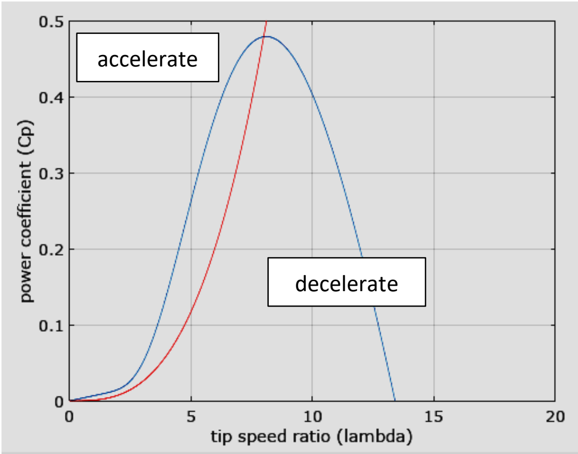
$$\frac{d\omega}{dt} < 0 \qquad when \qquad C_p < \left(\frac{C_{pMax}}{\lambda_{opt}^3}\right)\lambda^3 \qquad (13)$$

$$\frac{d\omega}{dt} > 0 \qquad when \qquad C_p > \left(\frac{C_{pMax}}{\lambda_{opt}^3}\right)\lambda^3 \qquad (14)$$

which gives the condition for rotor slow-down (13) and speed-up (14). We can show this graphically, in the plot below the blue curve is just $C_p(\lambda)$, how the power coefficient varies with tip-speed ratio. and the red curve is just

$$\left(\frac{C_{pMax}}{\lambda_{opt}^3}\right)\lambda^3$$

So, when the blue curve is greater than the red curve, the turbine speeds up, and when the blue curve is less than the red curve the turbine slows down.



4.8 Region 3 Pitch Controller

When the turbine has reached its rated rotational speed, for CART3 this is 3.952 rad/sec, then power needs to be shed (or at least not absorbed) if the wind speed increases any further. This is done by changing the pitch β (beta) of the turbine blades, so they extract less energy from the wind. Fig 4.4 shows how the pitch angle is defined, it is 0^o when the flat face of the blade is facing the wind, and is 90^o when the sharp edge is facing into the wind. Remember that in Region 2 the pitch is set to around 3.7^o so the blade is almost flat facing the wind. This may seem a little odd, but the truth is, even when the wind speed comes straight from the left, since the blade is rotating around the turbine hub, it ‘sees’ (or rather ‘feels’) the wind coming from an oblique angle. Also, since each place along the blade is moving with a different speed (fastest near the tip), then this felt wind angle is different. This is why turbine blades are not flat but are twisted.

The purpose of the Region 3 pitch controller is to change the pitch so it extracts less power from the wind as the wind speed increases. In Region 3 the turbine is operating at its designed specification, for CART-3 this is 600kW. Extracting any more power would cause the turbine to fail, and likely explode. A classical PID controller can be used here, though the derivative part is not often used since this is sensitive to abrupt changes. The formulation of the proportional part of the controller is straightforward, the difference between actual and rated rotation velocity is used to change the pitch angle,

$$\frac{d\beta}{dt} = -K_p(\omega_{rated} - \omega) \qquad (15)$$

So, if an increase in wind speed makes the turbine rotate too fast then $\omega > \omega_{rated}$ and the right hand side of (13) is positive so the blade pitch is increase, so there is less of the blade ‘facing’ the wind. When the turbine is rotating too slow, the right hand side is negative, so the pitch is decreased, and the blades absorb more power. The goal is quite straightforward, to ensure that the turbine rotates at its rated velocity

$$\omega = \omega_{rated} \qquad (16)$$