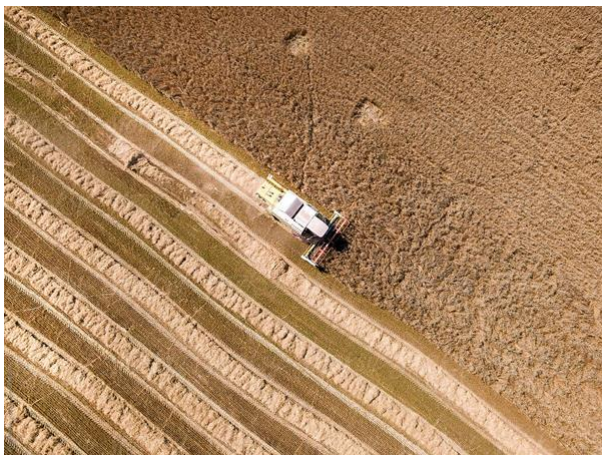


How a Farmer would calculate π

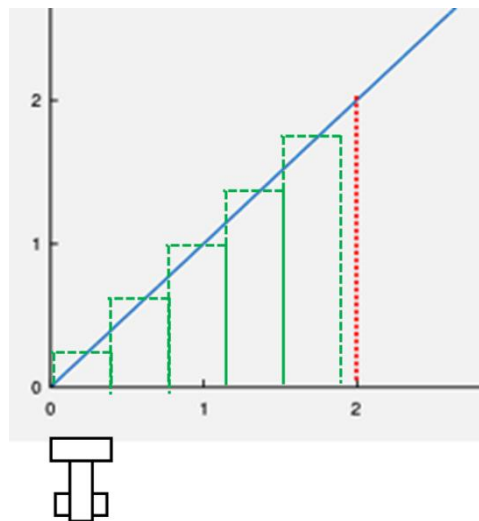
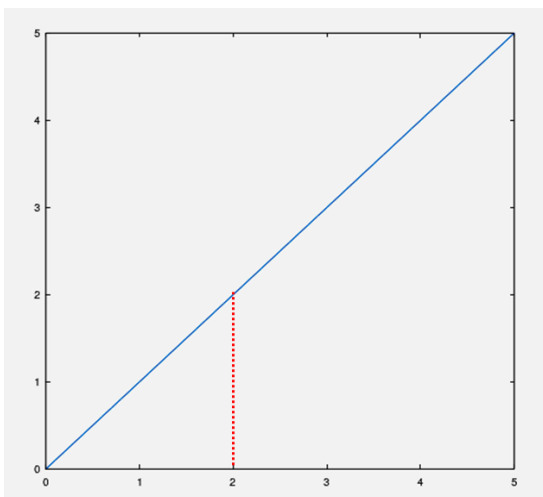
Here's an aerial view of a combined harvester at work. Perhaps someone from the 'Drones' module has been stalking someone from 'Nature'?



The point is the harvester is moving along a series of parallel strips. The farmer could note the total distance travelled as he harvests a whole field and multiplying this by the width of his strip would give him the total area of the field. Let's apply this to the problem of finding the area under a curve (function). It's called Numerical Integration.

Numerical Integration.

Here's a simple maths function $y = x$ and its graph is sketched below on the left. The question is, how to calculate the area under the graph from 0 to, say, 2.



We can do this by inspection. The area of the triangle up to the red line is **(width * height) / 2**. So, we get an area of 2 square units. Back to the farmer. If this was his field, he would drive up and down with his combine cutting out his strips like the diagram on the right. The areas of his strips would sum up like this

$(0.28 * 0.42) + (0.62 * 0.42) + (1.0 * 0.42) + 1.38 * 0.42) + (1.8 * 0.42)$ which sums to 2.13. So, the answer is an *approximation*, (close but not spot on) and we suspect that if the width of the harvester was smaller, then the error would get less. We shall see.

Mathematically, the process of finding an expression for the area under a curve (without counting, or ploughing) is called integration and it has the following symbols

$$A = \int_0^2 x dx$$

where the x inside is the function we are integrating. The result of the above is $\frac{1}{2}x^2$ which agrees with our triangle calculation above. So, what about the farmer? Well, he is approximating this exact integral by summing a series of strips like this

$$A = hx_1 + hx_2 + hx_3 + \dots + hx_N$$

which we can write using the Greek capital S (Σ) for sum,

$$A = \sum_i^N hx_i$$

where h is the width of his combine and x_i is the length of the i 'th strip which gets progressively longer.

The Farmer wants to find pi.

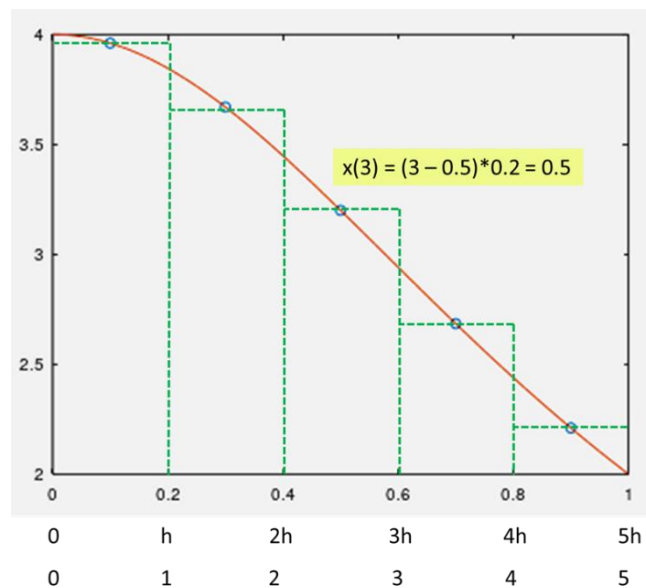
Here's an interesting function

$$\frac{4}{1+x^2}$$

and its integral is

$$\int_0^1 \frac{4}{1+x^2} dx$$

It turns out this integral is just pi! So, here's a plot of the function.



The farmer has chosen to reap the corn in five intervals. Since the field has width 1 unit, this means the width of each interval is $h = \frac{1.0}{5} = 0.2$. Let's agree to put the farmer to bed.

To calculate the area of each strip, we need to know its height. This is given to us by the function, when we provide the x -value for each strip. So, we need an expression for the x value of each strip. The yellow highlight shows how we calculate this for the third strip. The x -value at the end of the third strip is just $3h$ so the x -value at the centre of the third strip is just $(3 - 0.5)h$. The expression for the x -value in terms of the strip number i is therefore

$$x_i = h(i - 0.5)$$

The area of the i -th strip is the height of the strip multiplied by the strip width

$$a_i = \frac{4}{1 + x_i^2} h$$

Now we have enough information to write down our algorithm and express this in pseudo-code.

Algorithm and Code

Based on the above, the following algorithm emerges

- Choose N, the number of intervals on the range 0-1
- Calculate the strip size $1.0/N$
- Loop over the strips (index i)
 - Calculate the x-value $x = h * (i - 0.5)$
 - Calculate the strip height (function value for the x) $y = 4.0/(1 + x*x)$
 - Calculate the strip area $a = y*h$
 - sum the areas $A += a$

This can be easily translated into pseudo-code

```
user input N
h = 1.0/N;

for i = 1 to N
  x = h*( i- 0.5);
  y = 4.0/(1 + x*x);
  Area += y*h;
end for

printout Area
```

A template is provided which does the user input and printout. All you need to do is to code the **for** loop

Parallelization

Once you have got the above algorithm running, it's time to parallelize it. There is a **for** loop, so that's a hint. But most important is that you look for any potential data-races, a variable which is being subject to **read-update-write**. If you find such a variable, then you must deal with this according to what you have learned.

Investigations

The template provided asks you to input the value of **N** the number of intervals (strips). It outputs the value of **pi** computed and also the percentage error with the real value of pi. It also times the run. You could do an investigation where you change **N**. You could also change the number of threads. My machine has 16 threads, and I looked at a range of numbers of strips from 1000 to 100,000. What you do will depend on your machine.