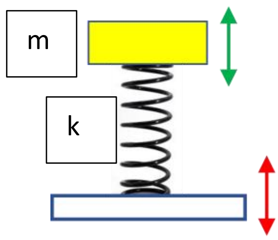
Piezo-electric Vibration Energy Harvester

These work on the property of a piezo-electric crystal. When it is bent, then it produces a voltage. This can be connected across an external load resistor to produce a current, so that electrical power is generated.

The following notes are based on a research paper by Dhakar et al., which is available to you. Some figures have been taken from this paper. There are many good illustrations in that paper which you should glance over.

This VEH is really quite simple. Shown on the left below, the pizeo crystal is sandwiched between two thin metal plates to capture the electric charge. The crystal is held fixed at the left, and the right hand end is free to vibrate so that the whole thing bends. You can see this in the model I created in UDK.

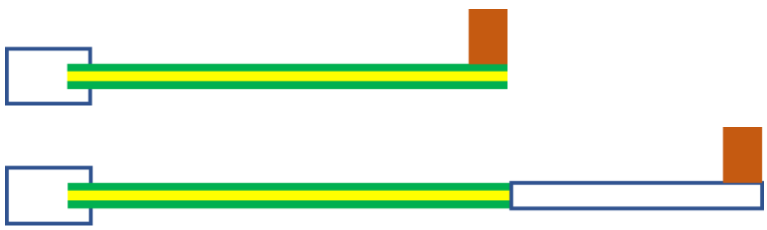
The system behaves the same as a mass *m* attached to a spring stiffness *k*, ah truck suspension again. This is shown in the middle. The expression for the natural frequency of oscillation is also given.



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

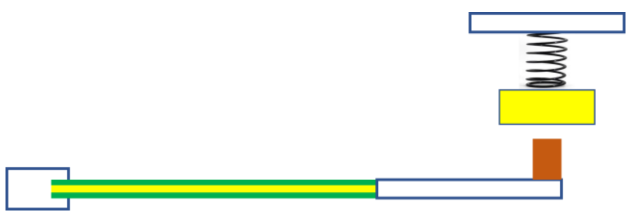
But there is a problem. When the mass and stiffness of the system are estimated, the natural frequency of oscillation turns out to be around 250Hz which is just too large, since the frequencies of potential energy sources such as household items, buildings, cars and human body motions are much much smaller.

This paper (and this mini-project) looks at how to reduce the natural frequency of the VEH. One way is to add some extra mass to the end of the beam and the other way is to add a second plastic beam onto the end of the piezo beam (which reduces the effective stiffness. These solutions are shown below.



The second solution, an additional plastic beam plus the extra mass is very effective, and reduces the natural frequency of vibration to around 25Hz which makes the device very useful. You can see this in Fig.10 of the Dhakar paper.

The authors also discuss quite a novel alternative. This involves restricting the range of movement of the mass at the end. There is an ‘end-stop’ which is a mass connected to a spring of greater stiffness than the beam system, see the diagram below. So when the beam mass (brown) hits the end-stop mass (yellow) then its stiffness increases.



To see the effect of this, let’s look at the author’s Fig.8. reproduced below.

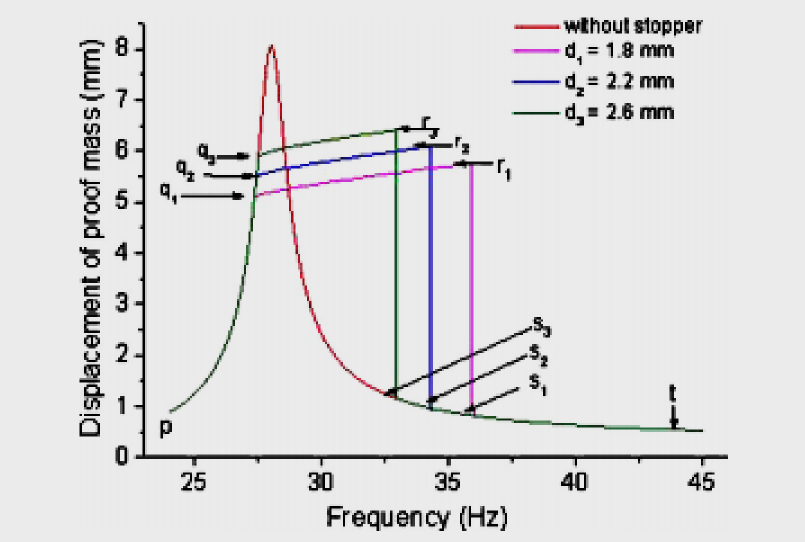


Fig. 8. Simulated frequency responses of EH with stopper at one side for *d* = 1.8 mm, 2.2 mm and 2.6 mm.

You can see the resonance curve reaching a peak at just above27.5 Hz then falling gracefully down. This is the curve labelled ‘without stopper’. The other curves show the effect of the stopper, placed at three different distances from the proof mass. The effect is amazing, the displacement of the mass is fairly constant over a large range of frequencies. Sure, it is not the peak, but it’s at 6/8 of the peak which is not bad. So this VEH can harvest energy from a wide range of oscillating systems.

The Mathematical Model

This is quite straightforward, it is just a linear oscillator, like the monster truck. The expressions are:

$$\frac{dz}{dt} = v$$

$$\frac{dv}{dt} = \frac{1}{m}(-kz - cv) + Acos(\Omega t)$$

Here, *z* is the vertical displacement at the end of the beam. But the frequencies involved in this experiment are just too large (25 – 45 Hz) to be useful in an UDK simulation. So we need to slow down the oscillations, say by a factor of 10. We can do this by *scaling* the above equations. We do this by replacing *t* by βt where the scaling factor $\beta = 10$. The above equations transform to

$$\frac{dz}{dt} = \frac{1}{\beta} v$$

$$\frac{dv}{dt} = \frac{1}{\beta}\frac{1}{m}(-kz - cv) + Acos(\Omega t/\beta)$$

and it’s these equations we code. You can see that since $\beta = 10$ is on the bottom of the rhs’s, then these are smaller by a factor 10 which slows everything down by this factor.