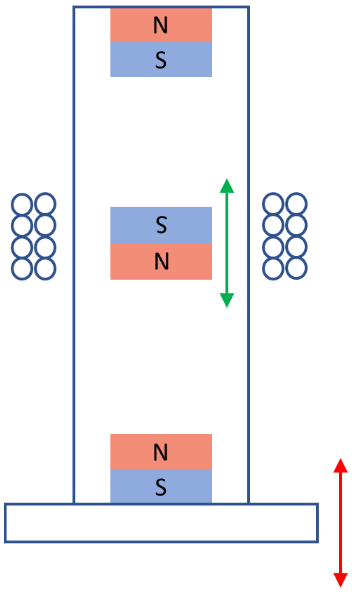
Magnetic Levitation Energy Harvesting

Energy harvesting from vibrating structures is a recent and active area of research. Many mechanical systems vibrate (car suspension, hair clippers, buildings, aircraft wings, your shoes when walking) and it could be useful to extract energy from these systems, in the form of electricity. One such system is the magnetic energy harvester shown below.



The top and bottom magnets are fixed inside a cylinder and arranged so the centre magnet, which is free to move, is levitated between them by repulsive forces.

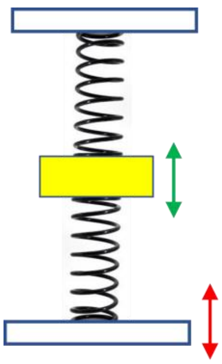
When the base vibrates, then the cylinder and top and bottom magnets oscillate, and this forces the centre magnet to oscillate, though there is relative motion between the centre magnet and the cylinder.

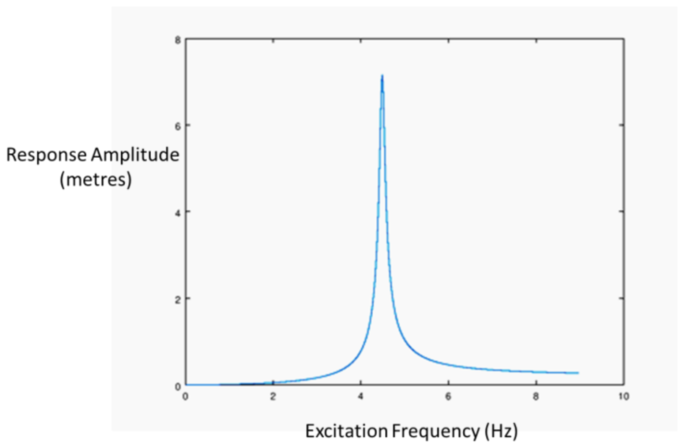
A coil of wire is placed as shown, so when the central magnet oscillates, it induces a voltage in the coil which can drive a current through a load resistor. So power is harvested.

The interesting feature of this system is the way the force on the central magnet changes with the displacement of this magnet from its levitated position. It turns out that this is non-linear and has the characteristics of a Duffing oscillator where the force is expressed as

$$F = -k_1 x - k_3 x^3$$

We are interested in how the amplitude of the voltage generated depends on the frequency of vibration of the base plate. Most structures vibrate with a particular frequency, so we are interested in 'tuning' our Vibration Energy Harvester (VEH) to this frequency. When the VEH frequency is the same as the structure frequency, then maximum energy is extracted. Consider the *linear* system shown below, designed to resemble the maglev.





For this linear system, the amplitude response is shown on the right, and peaks at one definite frequency, the natural frequency of oscillation of the VEH. The closer the structure oscillation frequency is to this, the more energy is harvested. We use this linear system as a comparison with the maglev.

The following discussion of the maglev is taken from a couple of papers authored by Krysztov Kecik (2018,2020) which are available to you. Many of the following graphs are attributed to these papers. The graphs presented below show the amplitude response of the maglev for various forcing frequencies. The latter are expressed in radians per second where $\omega = 2\pi f$ where f is the frequency in Hertz. Three curves are presented for increasing maximum amplitudes of excitation. The black curve for excitation amplitude 0.005 m has the same shape as the curve for the linear system shown above. There is a clear peak, and we conclude that the cubic non-linearity is not large in this situation. However at larger values, the curve folds over to the right.

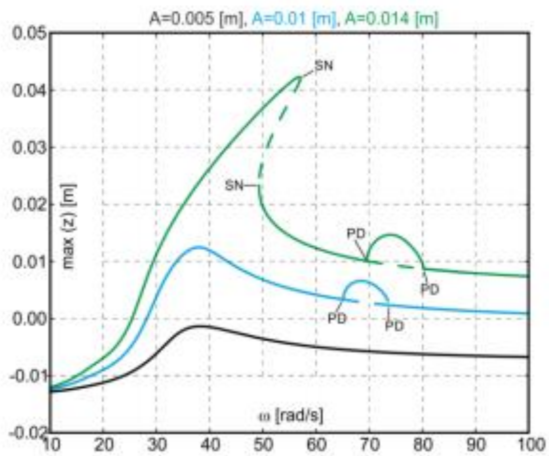


Fig. 5. Resonance curves of the magnet, for k=38.7N/m and R=2.3kΩ

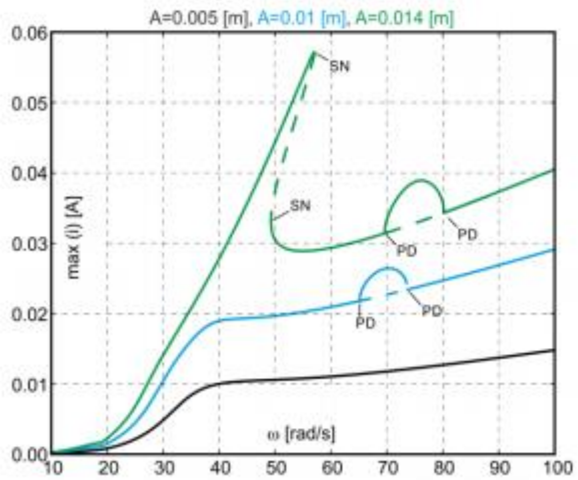
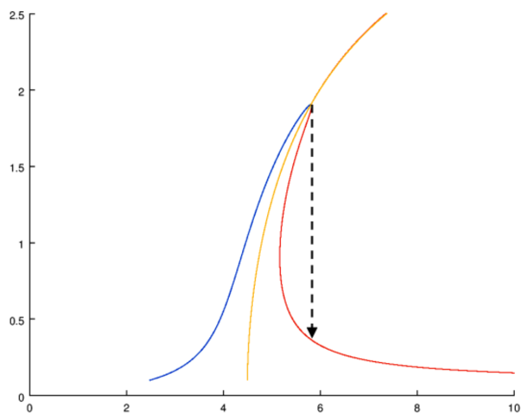


Fig. 6. Recovered current for the various excitation amplitude, for k=38.7N/m and R=2.3kΩ

This is very interesting for VEHs since it effectively extends the range of frequencies which produce a large response while the simple linear system responds over a narrower range of frequencies.

The Jump Phenomenon

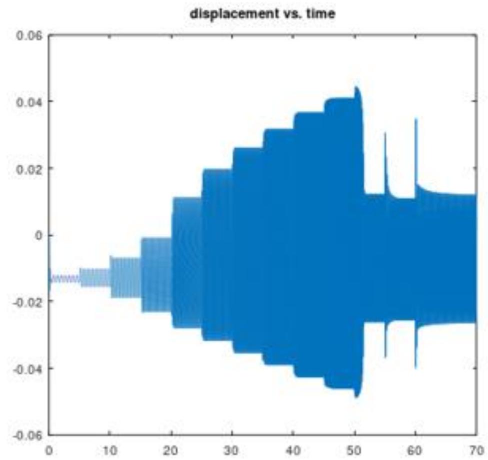


Starting with the lower frequency and slowly increasing the driving frequency the amplitude will increase along the blue line. When the cusp of the curve is reached, the amplitude jumps down to the red line, and this line is followed as the frequency is increased.

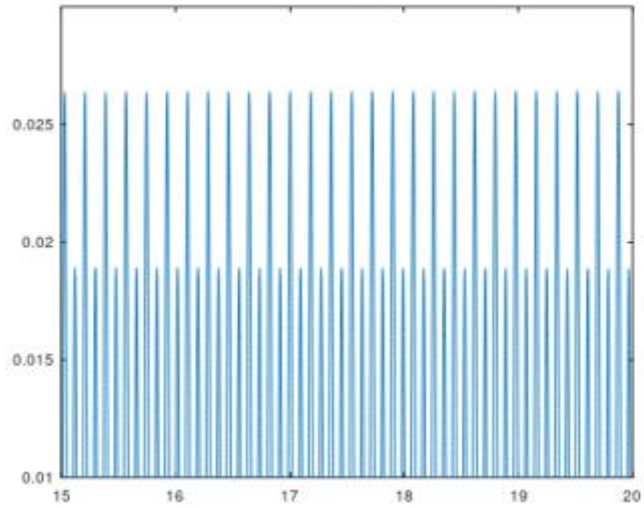
This can be investigated by progressively increasing the driving frequency in the code. An example is shown below.

Possible Investigations.

1) The first thing to do is to generate the response shown in Fig 5. This can be done by slowly increasing the frequency of oscillation in the code, rather than changing it by hand. The left plot below is the result of my experiments. You can clearly see the jump down. The envelope of this plot follows the green curve in Fig.5 almost perfectly.



k=38,7 Rtot = 2.3k A = 0.014 **Octave**



k=38,7 Rtot = 2.3k A = 0.01 omegaDrive = 70.0 Period-doubling

The loops on the response curve labelled PD show ranges of frequency where a period-doubling bifurcation has occurred.

2) The Kecik paper suggests some possible investigations. One very interesting one is to change the load resistance and see the effect of this on the power generated.

Maths Model

This is quite straightforward and consists of two parts, one is the mechanics of the oscillating magnet, the second is the induction of the voltage in the coil. Here's the magnet dynamics

$$\frac{dz}{dt} = v$$

$$\frac{dv}{dt} = \frac{1}{m}(-k_1 z - k_3 z^3 - cv - \alpha i) - g + \Omega^2 A sin(\Omega t)$$

and here's the induced current

$$\frac{di}{dt} = \frac{1}{L}\big(\alpha v - (R_C + R_L)\big)$$

If we look at the frequencies in Fig.5 $\omega = 40$ corresponds to about 6.4 oscillations per second. This is too large to visualize using UDK so we need to slow down the oscillations, say by a factor of 10. We can do this by *scaling* the above equations. We do this by replacing *t* by βt where the scaling factor $\beta = 10$. The above equations transform to

$$\frac{dz}{dt} = \frac{1}{\beta} v$$

$$\frac{dv}{dt} = \frac{1}{\beta}\left[\frac{1}{m}(-k_1 z - k_3 z^3 - cv - \alpha i) - g + \Omega^2 A sin(\Omega t)\right]$$

$$\frac{di}{dt} = \frac{1}{\beta}\frac{1}{L}\big(\alpha v - (R_C + R_L)\big)$$

and it's these equations we code. You can see that since $\beta = 10$ is on the bottom of the rhs's, then these are smaller by a factor 10 which slows everything down by this factor.