Robot – Body Dynamics and Motors

(1) Equations for the motor torque for a drive *u*

$$\tau_L = \frac{K}{R_w}(u_L V_s - K\omega_{ML}) - b_m \omega_{ML}$$

$$\tau_R = \frac{K}{R_w}(u_R V_s - K\omega_{MR}) - b_m \omega_{MR}$$

(2) But need to bump up due to the gearbox

$$\tau_L = \tau_L G$$
$$\tau_R = \tau_R G$$

(3) Equations for the robot body starting with acceleration forwards

$$a = \left(\frac{(\tau_L + \tau_R)}{r} - \ b_{lin} v\right)\frac{1}{m}$$

(4) Update forward velocity

$$\Delta v = a\Delta t$$
$$v = v + \Delta v$$

(5) And now angular acceleration

$$\alpha = \left(\frac{R(\tau_R - \tau_L)}{r} - \ b_{rot}\omega\right)\frac{1}{I}$$

(6) Calc angular vely and new angle

$$\Delta\omega = \alpha\Delta t$$
$$\omega = \omega + \Delta\omega$$
$$\Delta\theta = \omega\Delta t$$
$$\theta = \theta + \Delta\theta$$

(7) Calculation of robots x-y position using its angle (heading)

$$\Delta x = \ -v sin(\theta)\Delta t$$
$$\Delta y = \ v cos(\theta)\Delta t$$

(8) Final equations for the wheel angular velocities are

$$\omega_R = \frac{v + R\omega}{r}$$
$$\omega_L = \frac{v - R\omega}{r}$$

(9) Motor angular velocities are given by

$$\omega_{MR} = G\omega_R$$
$$\omega_{ML} = G\omega_L$$

Maths	Code	Meaning	
u_L	uL	drive signal applied to left motor by higher-level code	0.0
u_R	uR	same for right motor	0.0
ω_{ML}	omegaML	angular speed of left motor	
ω_{MR}	omegaMR	same for right motor	
τ_L	torqueL	torque provided by left motor	
τ_R	torqueR	same for right motor	
a	accelB	acceleration of the whole robot body	
v	velyB	velocity of the robot	0.0
x	xB	x-location of robot	
y	yB	y-location of robot	
α	alphaB	angular acceleration of robot body	
ω	omegaB	angular velocity of body	0.0
θ	thetaB	heading of the robot	0.0
ω_L	omegaL	angular speed left motor	0.0
ω_R	omegaR	angular speed right motor	0.0

Parameters

V_s	vSource	20.0	Battery Voltage
K	K	0.1	Electromotoric Constant
R_w	RWinding	2.0	Motor winding resistance
b_m	motorDamp	0.002	Damping inside motor
G	transRatio	2	Gearbox transfer ratio
r	wheelRad	0.05	Radius of the wheels
b_{lin}	linearDamp	0.1	Linear damping (friction) on robot body
m	massR	1.25	mass of the robot
R	robotRad	0.05	Radius of the robot (half wheel-base)
b_{rot}	rotDamp	1.35	Rotation damping (friction) on robot
I	IRobot	0.55	Moment of inertia of robot body

(10) Code the Robot PID Controller

$$e_{t-2} = e_{t-1}$$
$$e_{t-1} = e_t$$
$$e_t = s/10$$

$$\Delta u = K_p(e_t - e_{t-1}) + K_I e_t \Delta t + \frac{K_D}{\Delta t}(e_t - 2e_{t-1} + e_{t-2})$$

$$u_t = u_{t-1} + \Delta u$$
$$u_L = b - u_t$$
$$u_R = b + u_t$$

Variables

Maths	**Code**	**Meaning**	**ICs**
s	getSensorReading();	reading from the sensor	n/a
e_t	error	error at this time step	
e_{t-1}	e1	error from previous time step	0.0
e_{t-2}	e2	error two time steps ago	0.0
u_L	uL	controller drive to left motor	
u_R	uR	controller drive to right motor	
u_t	u	output of controller signal	0.0
Δu	deltaU	change in controller signal	0.0
b	baseDrive	constant drive to motors	1.0

Parameters

K_p	Kp	3.0	Proportional	Controller
K_d	Kd	0	Derivative	
K_i	Ki	0	Integral	
	sensorOffset	1.0	distance between each sensor	