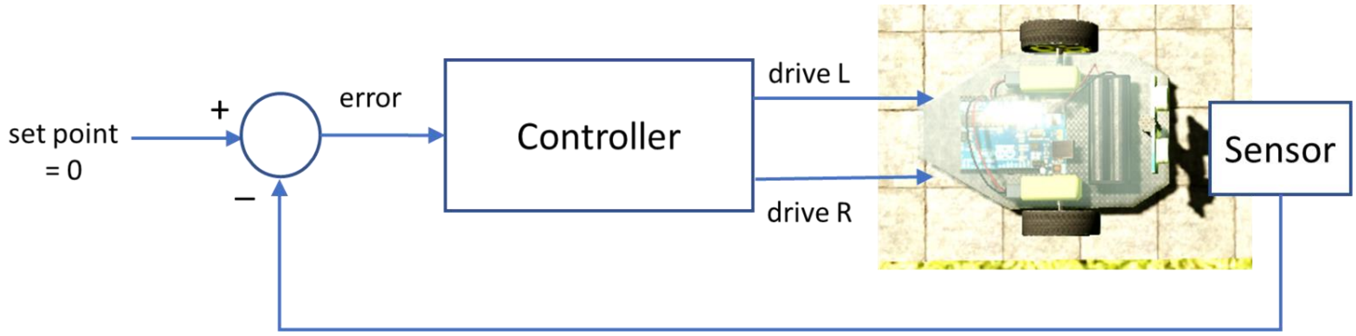
Introduction to Differential Drive Robots

Here's a snip of the line-following robot. There are two wheels which rotate with differing speeds to make the robot move forwards or in a curved arc to the left or right. This differential motion is used to keep the robot on the line. You can see the sensor array pointing down, those colored blue have detected the line, and those colored red have missed the line. The middle of the blue gives an indication of where the line is, ideally the blue should be at the centre of the sensor array. So any difference from the sensor gives the robot an error signal which it can use to correct its position by adjusting the speed of its wheels.

Here's a diagram of the robot and its controller. You can see that the controller has two outputs, to the motors of both wheels.



Let's see how the system works and therefore the code we must write. All symbols are explained at the end of this document. The maths model for the motors looks like this. The inputs to the motors from the controller are u_L and u_R, and each motor responds by providing a torque τ_L and τ_R. Details of these expressions will be provided later

$$\tau_L = \frac{K}{R_w}(u_L V_s - K\omega_{ML}) - b_m \omega_{ML}$$

$$\tau_R = \frac{K}{R_w}(u_R V_s - K\omega_{MR}) - b_m \omega_{MR}$$

The torques produce a linear acceleration of the robot making it move along a straight line

$$a = \left(\frac{(\tau_L + \tau_R)}{r} - b_{lin}v\right)\frac{1}{m}$$

and also an angular acceleration making it rotate around its centre

$$\alpha = \left(\frac{R(\tau_R - \tau_L)}{r} - b_{rot}\omega\right)\frac{1}{I}$$

So the linear acceleration changes the forward velocity

$$\Delta v = a\Delta t$$
$$v = v + \Delta v$$

and the angular acceleration gives an angular velocity which rotates the robot through an angle

$$\Delta\omega = \alpha\Delta t$$
$$\omega = \omega + \Delta\omega$$
$$\Delta\theta = \omega\Delta t$$
$$\theta = \theta + \Delta\theta$$

The actual x-y displacement of the robot is calculated from its forward velocity and its current angle

$$\Delta x = -v\sin(\theta)\Delta t$$
$$\Delta y = v\cos(\theta)\Delta t$$

A little more detail

There is a gearbox between motors and wheels, motors rotate faster than wheels. The gearbox ratio *G* serves to bump up the torque from motors to wheels

$$\tau_L = \tau_L G$$
$$\tau_R = \tau_R G$$

Also, we need to calculate the motor speeds. These are

$$\omega_R = \frac{v + R\omega}{r}$$
$$\omega_L = \frac{v - R\omega}{r}$$

and finally we use the gearbox ratio to link motor and wheel speeds

$$\omega_{MR} = G\omega_R$$
$$\omega_{ML} = G\omega_L$$

The Motor Equation

A motor is driven by a voltage V which produces a current i in its windings. It is this current that determines the torque like this

$$\tau_m = Ki$$

where $K$is a constant. Since $i = V/R$ where R is the resistance of the motor windings we would expect the equation for torque to look like this

$$\tau_m = K\frac{V}{R}$$

but there is another factor involved. As the motor turns it produces a voltage with the opposite sign to V which we must subtract from V. The magnitude of this is

$$K\omega_m$$

where ω_m is the angular velocity of the motor. So the expression for the torque due to the driving voltage is

$$\tau_m = K\frac{(V - K\omega_m)}{R}$$

Finally there is an additional torque on the motor shaft from the damping (friction) inside the motor. The magnitude of this is $-b_m\omega_m$. So the total torque ***delivered*** by the motor to the ride is

$$\tau_m = \frac{K}{R}(V - K\omega_m) - b_m\omega_m$$

Robot motors are not driven with the full battery voltage V; to control the motors, a varying voltage is sent to the motors. This is the control signal or “drive” and is computed by the controller. The “drive” is represented by the symbol u which takes on a value between 0 and 1. So, if V_s is the battery (source) voltage, then the actual voltage delivered to the motor is

$$V = uV_s$$

So the final equation for the torque on the motor becomes

$$\tau_m = \frac{K}{R}(uV_s - K\omega_m) - b_m\omega_m$$

Maths	Code	Meaning	
u_L	uL	drive signal applied to left motor by higher-level code	0.0
u_R	uR	same for right motor	0.0
ω_{ML}	omegaML	angular speed of left motor	
ω_{MR}	omegaMR	same for right motor	
τ_L	torqueL	torque provided by left motor	
τ_R	torqueR	same for right motor	
a	accelB	acceleration of the whole robot body	
v	velyB	velocity of the robot	0.0
x	xB	x-location of robot	
y	yB	y-location of robot	
α	alphaB	angular acceleration of robot body	
ω	omegaB	angular velocity of body	0.0
θ	thetaB	heading of the robot	0.0
ω_L	omegaL	angular speed left motor	0.0
ω_R	omegaR	angular speed right motor	0.0

Table of Variables

V_s	vSource	20.0	Battery Voltage
K	K	0.1	Electromotoric Constant
R_w	RWinding	2.0	Motor winding resistance
b_m	motorDamp	0.002	Damping inside motor
G	transRatio	2	Gearbox transfer ratio
r	wheelRad	0.05	Radius of the wheels
b_{lin}	linearDamp	0.1	Linear damping (friction) on robot body
m	massR	1.25	mass of the robot
R	robotRad	0.05	Radius of the robot (half wheel-base)
b_{rot}	rotDamp	1.35	Rotation damping (friction) on robot
I	IRobot	0.55	Moment of inertia of robot body

Table of Parameters