Skymaster (without motor and gearbox). Advanced work – Period doubling Bifurcation

Here we shall follow the slides from Jen-Hao Yeh (link provided). A slightly different notation is used. This page explains how to move from our notation to the new. The results are found in the table at the bottom of the page.

The Skymaster with motor and gearbox can be represented by the following differential equation

$$mL^2\ddot{\theta} + b_a\dot{\theta} + mgLsin(\theta) = \frac{K}{R_w}\left(A_d\cos{(\omega t)} - G\dot{\theta}\right) - b_m\dot{\theta}$$

Removing the motor and gearbox results in the direct forcing by the term $A_d\cos{(\omega t)}$ so we have

$$mL^2\ddot{\theta} + b_a\dot{\theta} + mgLsin(\theta) = A_d\cos{(\omega t)}$$

and after a slight re-arrangement, we have

$$\ddot{\theta} + \frac{b}{mL^2}\dot{\theta} + \frac{g}{L}sin(\theta) = \frac{A_d}{mL^2}\cos{(\omega t)}$$

This can be written in standard form

$$\ddot{\theta} + 2\beta\dot{\theta} + \omega_0^2 sin(\theta) = \gamma\omega_0^2\cos{(\omega t)}$$

which is used in the Yeh presentation (link provided) as well as in John Taylor's *Classical Mechanics* (used by Yeh) and this is available online/

Yeh and Taylor use the following values of coefficients to explore the behaviour of this nonlinear system, and the equivalents for the Skymaster parameters are also given

Taylor/Yeh		UDK - Skymaster
$\omega = 2\pi$	Forcing frequency	Take *m=1* and *g = 9.8*, then
$\omega_0 = 1.5\omega$	Free-oscillation frequency $\sqrt{g/L}$	
$\beta = \omega_0/4$	Damping coefficient	$L = g/\omega_0^2$ $b = 2mL^2\beta$ $A_d = \gamma\, mL^2\omega_0^2$

All these coefficients/parameters are fixed; experimental investigations involve changing the value of γ which is the magnitude of forcing the pendulum.

The Duffing Equation

You will certainly come across reference to this equation for our forced pendulum. This is a driven-damped nonlinear oscillator with cubic damping. How is this related to the above?

Well remember that $sin\theta \approx \theta - \frac{1}{6}\theta^3$ (Taylor series) so the above equation becomes

$$\ddot{\theta} + 2\beta\dot{\theta} + \omega_0^2\left(\theta - \frac{1}{6}\theta^3\right) = \gamma\omega_0^2\cos{(\omega t)}$$