Inverted Pendulum (Segway)

(1) Calculate the angular acceleration of the pendulum and its new angle

$$\alpha = \frac{(m_P + m_C)g\theta + u + bv_x}{m_C l}$$

(2) Update the angular velocity and angle

$$\Delta\omega = \alpha\Delta t$$
$$\omega = \omega + \Delta\omega$$
$$\Delta\theta = \omega\Delta t$$
$$\theta = \theta + \Delta\theta$$

(3) Calculate the linear acceleration of the cart

$$a_x = \frac{-m_P g\theta - u - bv_x}{m_C}$$

(4) Update its velocity and displacement

$$\Delta v_x = a_x\Delta t$$
$$v_x = v_x + \Delta v_x$$
$$\Delta x = v_x\Delta t$$
$$x = x + \Delta x$$

(5) PID-Controller equations: save the prior errors and calculate the current error

$$e_{t-2} = e_{t-1}$$
$$e_{t-1} = e_t$$
$$e_t = \theta_{des} - \theta$$

(6) Derive the drive signal

$$\Delta u = K_p(e_t - e_{t-1}) + K_I e_t \Delta t + \frac{K_D}{\Delta t}(e_t - 2e_{t-1} + e_{t-2})$$

$$u_t = u_{t-1} + \Delta u$$

Variables

Math	Code		ICs
Δt	deltaT		
α	alpha	angular acceleration of the pole	
ω	omega	angular speed of the pole	0.0
θ	theta	angle of the pole	
a_x	accelX	acceleration of the cart	
v_x	velyX	velocity of the cart	0.0
x	dispX	position of the cart	0.0
e_t	error	error at this time step	
e_{t-1}	e1	error from previous time step	0.0
e_{t-2}	e2	error two time steps ago	0.0
u	u	controller “drive”	

Parameters

Math	Code	Default	Meaning	
m_P	massPole	0.2	mass of the pole	
m_C	massCart	0.5	mass of the cart	
g	gravity	9.81		
l	len	0.3	length of pole	
b	damping	0.0	damping of cart	
K_p	Kp	10	Proportional	Controller
K_d	Kd	1	Derivative	
K_i	Ki	0	Integral	
θ_{des}	desTheta	0.0	desired pole angle	