Comp 3302 Nature of Computing

Boolean Algebra – Part 2

Color Coding for this Worksheet	
	Information
	Instructed Learning
	Challenges

Activities – Checking. Identities from Truth Tables.

Example how to check the identity

$$X + 0 = X$$

a) Start with the truth table

X		
0	0	0
0	1	1
1	0	1
1	1	1

b) Now highlight rows for two different value of X (0 and 1) where the middle column is 0

X		
0	0	0
0	1	1
1	0	1
1	1	1

c) Check that the third column in the highlighted row is *X*. This proves the identity

1. Use the above technique to prove that

a) $X + 1 = 1$
b) $X + X = X$
c) $X + \bar{X} = 1$

2. Now apply the above technique to the AND gate to show that

a) $X.1 = X$
b) $X.0 = 0$
c) $X.X = X$
d) $X.\bar{X} = 0$

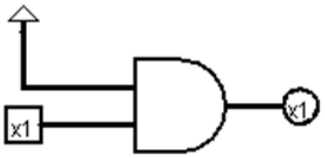
These results are very important in applying Boolean algebra

Activities – Testing some Boolean Identities.

1 Build the following circuit to test the identity rule

$$X.1 = X$$

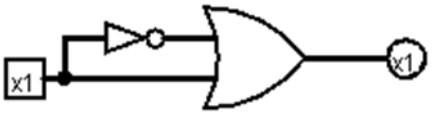
which says that the input when and-ed with a "1" remains unchanged.



2 Build the following circuit to test the complement rule

$$X + \bar{X} = 1$$

which says that a value when or-ed with its inverse is always true.



3 Build the following circuit to test the absorption rule

$$X + X.Y = X$$

This states that if you have a boolean variable *X* and you or it with "*X and-ed* with any other variable *Y",* then *X* remains unchanged.

The input X is at the bottom and Y is at the top.

4 Build two circuits to test "De-Morgan's" rule and show that it works.

5 **Applying Boolean Algebra to simplify Boolean expressions.**

Use Boolean algebra to prove the following simplifications. Show all steps of your working.

Let's start with an example proving that

$A.\bar{C} + AB\bar{C} = A.\bar{C}$

Here are the steps.

1	$A.\bar{C} + AB\bar{C} = A.(\bar{C} + B\bar{C})$	distributive
2	$A.(\bar{C} + B\bar{C}) = A.\bar{C}(1 + B)$	distributive

3	$A.\bar{C}(1+B) = A.\bar{C}.1$	identity
4	$A.\bar{C}.1 = A.\bar{C}$	identity

Off we go. Set out all your proofs as in the above example.

(a) $A.\bar{A} + B = B$

(b) $A.B + A.C = A.(B + C)$

(c) $A.B + A.\bar{B} = A$

(d) $A.B + A.\bar{B} + A.C = A$

(e) $A + A.B = A$

(f) $\bar{B}.B.C + \bar{B}.B.A = 0$

(g) $\bar{A}.B + \bar{A}.\bar{B} = \bar{A}$

(h) $(\bar{A} + B).(A + B) = B$

(i) $(A + B).(A + C) = A + B.C$ *This is a **hard** proof.*

(j) $A.B.C + A.B.\bar{C} + A.\bar{B}.C + A.B.C = A.(B + C)$ ***Hard.***

(k) $C + \overline{B.C} = 1$ (Hint: DeMorgan)

(l) $\overline{A.B}(\bar{A} + B).(\bar{B} + B) = \bar{A}$

6 **Challenges – All gates can be made from NAND gates**

(a) Prove, using DeMorgan that a NOT gate can be made from a single NAND gate. Sketch the circuit.

(b) Prove that an AND gate can be made from two NAND gates. Sketch the circuit. You will need your result of (a)

(c) Prove that an OR gate can be made from three NAND gates. You will need your previous results.